

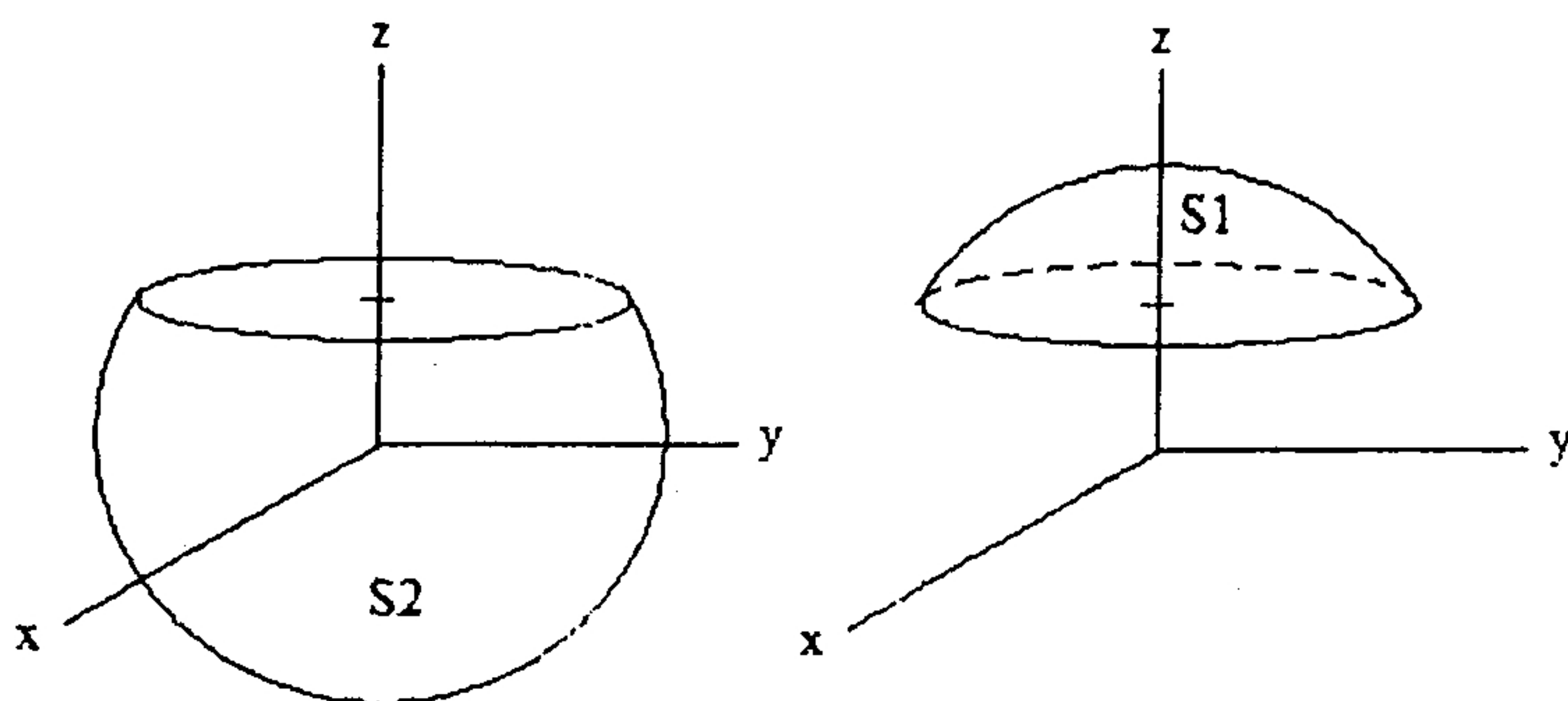
國立台灣科技大學九十七學年度碩士班招生試題

新組別：機械工程系碩士班乙組、丙組、丁組

目：工程數學

總分100分

1. A spherical solid $x^2 + y^2 + z^2 \leq 100$ is cut into two pieces (S1 and S2) by a plane $z = 5$ as the figure below. Determine the volume of the small piece S1. (20%)



2. Solve the following differential equation.

$$x y' = -2y + \sin x \quad (20\%)$$

3. Solve the following initial value problem.

$$x^2 y'' + 2x y' + 100.25 y = 0, \quad y(1) = 2, \quad y'(1) = -11. \quad (20\%)$$

4. z is a complex variable, and $i = \sqrt{-1}$,

(a) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{i^n}{2^{n+1}} (z + 5i)^n. \quad (10\%)$$

(b) Find all values of $(-1 + i)^{-3i}$.

(10%)

5. Solve the boundary value problem

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \quad (0 < x < 1, t > 0)$$

$$y(0, t) = y(1, t) = 0 \quad (t > 0)$$

$$y(x, 0) = 0 \quad (0 < x < 1)$$

$$\frac{\partial y}{\partial t}(x, 0) = 1 \quad (0 < x < 1)$$

(20%)