

國立台灣科技大學九十七學年度碩士班招生試題

所組別： 自動化及控制研究所碩士班甲組、乙組

目： 工程數學

總分為100分，題號請標示清楚。

1. Solve the following differential equations:

$$(1) \quad yy' + 2t + y^2 = 0; \quad y(0) = 2 \quad (10\%)$$

$$(2) \quad y'' - y = 5\sin^2(t); \quad y(0) = 2, y'(0) = -4 \quad (10\%)$$

2. Solve $x(t)$ and $y(t)$ from the simultaneous differential equations. (15%)

$$x' - x + y = e^{-t}$$

$$y' - 2x + 2y = e^{-t} \sin(2t)$$

3. Solve the following equation by using Laplace Transform method. (15%)

$$y' + 3y + 2 \int_0^t y dt = H(t - 2)$$

$$\text{where } y(0) = 1 \quad \text{and} \quad H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

4. Determine eigenvalues and eigenvectors of A (20%)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

5. Find $\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}$ (10%)6. Solve the P.D.E. $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ with the following boundary conditions:

$$\phi(0, y) = 0$$

$$\phi(h, y) = 0$$

$$\phi(x, 0) = g(x)$$

$$\phi(x, l) = 0$$

(20%)