國立雲林科技大學
97學年度碩士班入學招生考試試題

系所：財金系、企管系
科目：微積分

1. **Evaluate the limit, if it exists:**

(a) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$. (5%) (b) $\lim_{x\to 0^+}\frac{\int_0^{\sqrt{x}}\sin\left(t^2\right)dt}{\sin\left(x^{3/2}\right)}$. (5%)

2. **Find the equation of tangent to the curve of the graph of**

$8\left(x^2+y^2\right)^2=100\left(x^2-y^2\right)$ **at the point** $P(3,-1)$. (10%)

3. **Evaluate the surface integral** $\iint_G y^2z^2\,dS$ **, where** G **is the part of the cone**

$z=\sqrt{x^2+y^2}$ **between the planes** $z=1$ **and** $z=2$. (10%)

4. **A manufacturer of model airplane engines finds that it takes** L **units of labor and** C **units of capital to produce** $f(L,C)=\beta+\frac{2}{3}\ln L+\frac{1}{3}\ln C$ **units of the product. If a unit of labor cost \$100 and a unit of capital costs \$200 and \$150,000 is budgeted for production, determine how many units should be expended on labor and how many units should be expended on capital in order to maximize production.** (10%)

5. **Determine the radius of convergence and the interval of convergence of** $\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{2k+1}$. **Moreover, find the sum of** $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$. (10%)

6. **Evaluate** $\lim_{n\to\infty}\left\{\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+n}\right\}$ (10%)

7. **Find a polynomial f of lowest possible degree such that**

$f(x_1)=a_1, f(x_2)=a_2, f'(x_1)=b_1, f'(x_2)=b_2$, **where** $x_1\neq x_2$, **and** a_1, a_2, b_1, **and** b_2 **are given real numbers.** (10%)

8. **The following functions F and G are defined for all real** x **by the equations given. Give the domain of composite function** $G\circ F$ **and a formula for** $(G\circ F)(x)$. $F(x)=x+5$, $G(x)=\frac{|x|}{x}$, **if** $x\neq 0$, $G(0)=1$. (10%)

9. **Let** $f:S\to T$ **be a function. If** A **and** B **are arbitrary subsets of** S**, prove that**

$f(A\cup B)=f(A)\cup f(B)$ **and** $f(A\cap B)\subseteq f(A)\cap f(B)$. (10%)

10. **Prove that** $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx=\frac{\pi}{4}$. (10%)