



1. Find the general solution of the differential equation (10 分)

$$[D^3 - 2D^2 + D]y = 2x; \quad [\text{Note: } D^n y = y^{(n)} = \frac{d^n}{dx^n} y]$$

2. Find the general solution, $y(x) = c_1 y_1(x) + c_2 y_2(x)$, of the differential equation $x^2 y'' + xy' - y = 0$, $x > 0$. To explain if y_1, y_2 are linear independent by Wronskain test. (15 分)

3. Let $A = \begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix}$, find: (1) P , and diagonal matrix $D = P^{-1}AP$,
(2) $(A^2 + 6A + 4I)^5 = ?$ (10 分)

4. To solve the initial value problem, $\begin{cases} x_1' = 2x_1 - 10x_2 \\ x_2' = -x_1 - x_2 \end{cases}$, $X(0) = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$, by matrix methods. (15 分)

5. Find the Laplace transform for the following functions (10%)
 $[\sin(t-1) + (t^2 - 2)]H(t-1)$

6. Find the inverse Laplace transform for the following functions.

(a) $\ln[(s+2)/(s-1)],$ (10%)

(b) $\frac{se^{-2s}}{(s+2)^2(s^2+4s+8)},$ (10%)

7. Find the sum of the series $\sum_{n=1}^{\infty} (-1)^n / (4n^2 - 1)$. (hint :expand $\sin(x)$ in a Fourier cosine series on $[0 \ \pi]$ and choose an appropriate value of x . (10%)

8. Find the inverse Fourier transform for function: $\frac{2e^{(\omega-2)i}}{[2 + (\omega-2)i]}$. (10%)