



乙理

共 7 題，合計 100 分，請依序作答，否則不計分。

1. A discrete system is described by the dynamic equation (state equations and output equation)

$$\begin{aligned} \mathbf{x}(k+1) &= \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \\ y(k) &= [1 \quad 1] \mathbf{x}(k) \end{aligned}$$

(i) Find the state transition matrix $\Phi(k) = ?$ (10%)

(ii) Find the solution $y(k)$ if $\mathbf{x}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $u(k) = 1, k \geq 0$. (10%)

2. Find the input-output differential equation to describe the *RLC* network in Figure 1, where $u(t)$ and $y(t)$ are the input voltage source and output voltage of *C*, respectively. (15%)

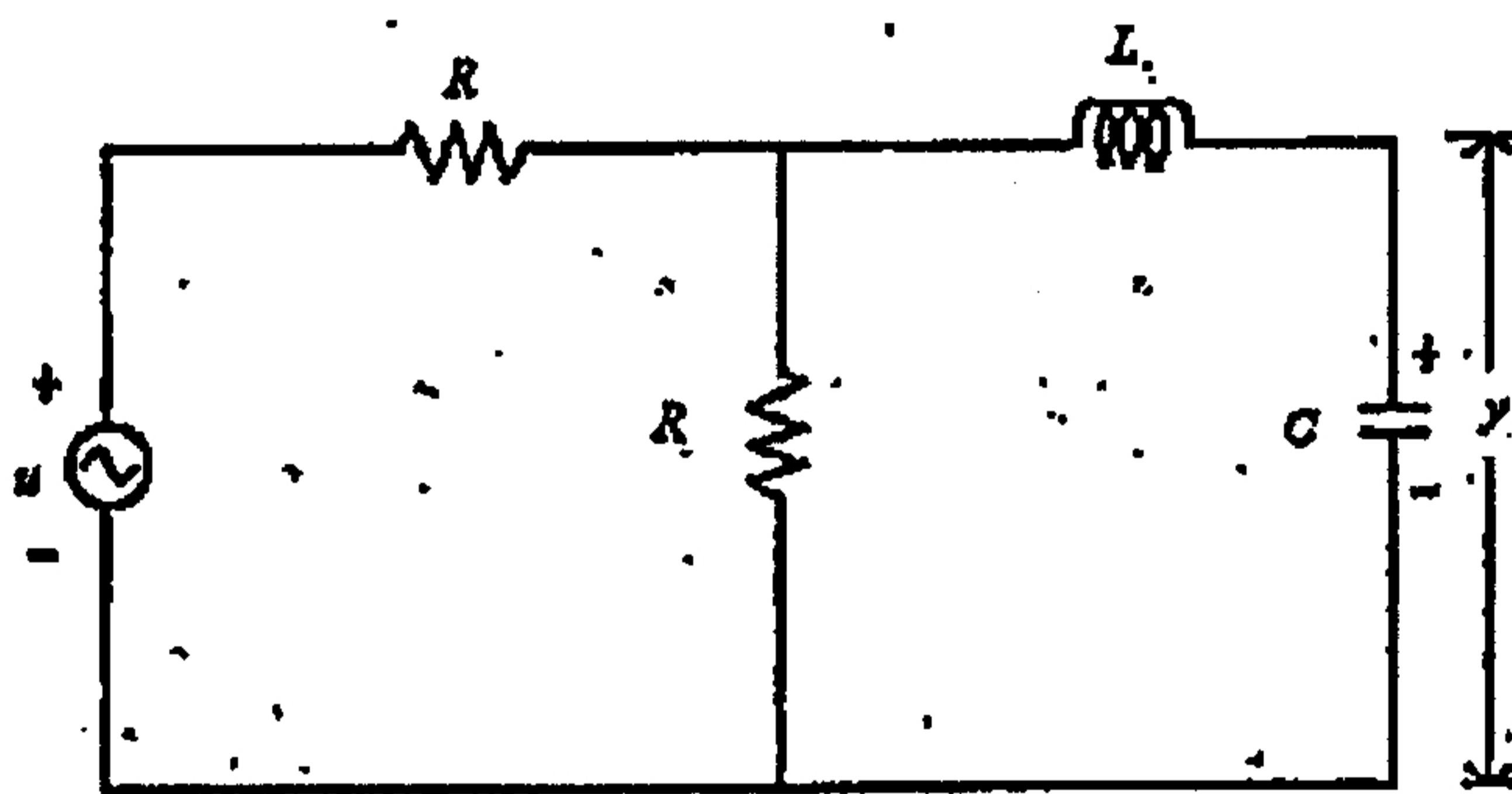


Figure 1

3. Consider a linear time-invariant system of the closed loop transfer function

$$G_0(s) = \frac{Y(s)}{U(s)} = \frac{2}{s+1}, \text{ where } u(t) \text{ and } y(t) \text{ denote the input and the output, respectively. Find the steady state responses } y(\infty) \text{ when } u(t) = 1 + 2\cos(t).$$

respectively. Find the steady state responses $y(\infty)$ when $u(t) = 1 + 2\cos(t)$.

(15%)



4. For the transfer function below, find how many poles are in the right half-plane, and on the $j\omega$ -axis. (10%)

$$T(s) = \frac{K}{s^7 + s^6 + 2s^5 + 2s^4 - s^3 - s^2 - 2s - 2}$$

5. Given the unity feedback system shown as Figure 2, where

$$G(s) = \frac{K}{(s+2)(s^2+2s+2)}$$

solve the following problem: (15%)

- (a) Sketch the Nyquist diagram (mapping only the positive $j\omega$ -axis). (10%)
 (b) Find the gain margin with $k=10$. (5%)

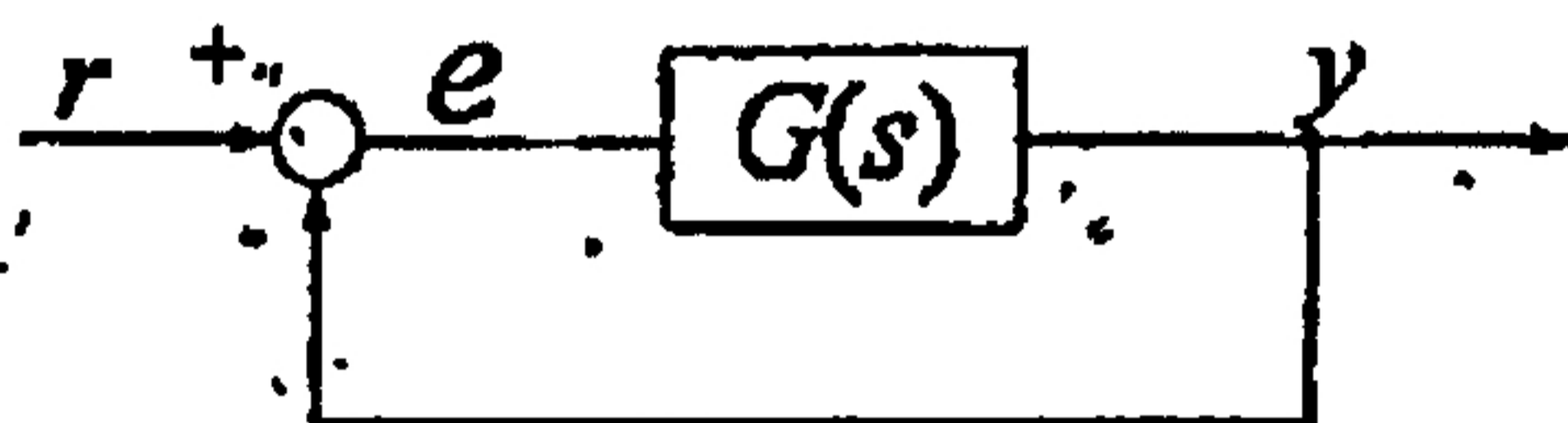


Figure 2.

6. Consider the dynamical system, $\frac{dx}{dt} = Ax(t) + bu(t)$,

$$\text{where } A = \begin{bmatrix} 0 & -1 \\ 5 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Find the control $u(t) = -kx(t)$, such that the desired poles are located on $-1 \pm j1$. (15%)

7. Check the controllability and observability of

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} u(t) \quad (10\%)$$

$$y(t) = [1 \ 0 \ 0]x(t)$$