國立彰化師範大學97學年度碩士班招生考試試題

系所：資訊工程學系碩士班　　　　　　　　科目：離散數學及線性代數

☆☆請在答案紙上作答☆☆　　　　　　　　

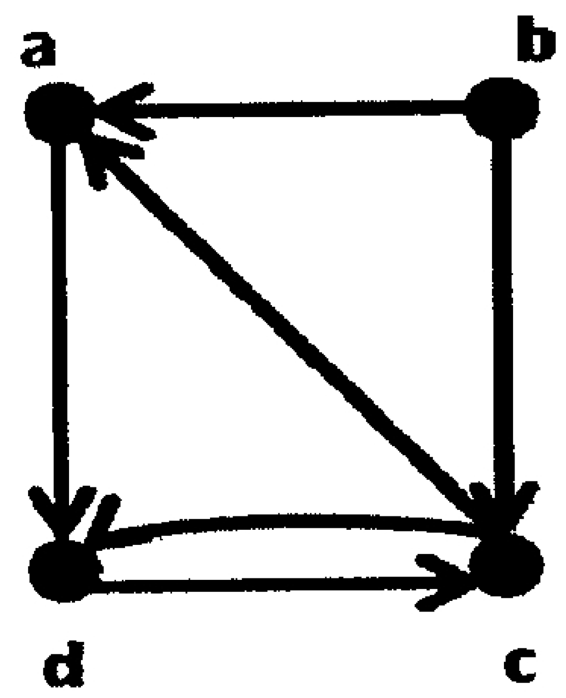
1. Solve the recurrence relation $a_n-3a_{n-1}=5\times3^n$, where $n\geqq 1$ and $a_0=2$. (10%)

2. A *coloring* of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color. The *chromatic number* of a graph is the least number of colors needed for a coloring of this graph. Let *m*, *n* are integers. (a)What is the chromatic number of *complete graph* K_n? (b) What is the chromatic number of the *complete bipartite graph* $K_{m,n}$? (c) What is the chromatic number of *cycle graph* C_n, $n\geqq 3$? (d) What is the chromatic number of *n-cube graph* Q_n, $n\geqq 3$? (e) What is the chromatic number of *wheel graph* W_n, $n\geqq 3$? (10%)

3. Let $m\in Z^+$ with *m* odd. Prove that there exists a positive integer *n* such that *m* divides 2^n-1. (10%)

4. Since an *equivalence relation* on a set includes a partition of that set, for $n\geqq 2$, *congruence modulo n (mod n)* partitions **Z** into the *n equivalence classes* $[0]=\{\ldots, -2n, -n, 0, n, 2n, \ldots\}$, $[1]=\{\ldots, -2n+1, -n+1, 1, n+1, 2n+1, 3n+1, \ldots\}$, …, $[n-1]=\{\ldots, -n-1, -1, n-1, 2n-1, 3n-1, \ldots\}$. Let Z_n denote the set $\{[0], [1], \ldots, [n-1]\}$. Find the set of *x* such that **25*x* mod 72=1.** (5%)

5. Let ***R*** be the relation with directed graph shown in Figure 1. Let *a*, *b*, *c*, *d* be a listing of the elements of the set. Use the ***Warshall's Algorithm*** to find the matrix of the *transitive closure* of ***R***.



(5%)

Figure 1

6. How many paths of length four are there from ***c*** to ***d*** in the graph in Figure 1? (5%)

7. Construct a *nondeterministic finite-state automaton* that recognizes the language generated by the regular grammar $G=(V, T, S, P)$, where $V=\{0, 1, A, S\}$, $T=\{0, 1\}$, and the productions in ***P*** are **S→1A, S→0, S→λ, A→0A, A→1A,** and **A→1.** (5%)

8. Let V be R^3 and let $S=\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $T=\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be bases for R^3, where $\mathbf{v}_1=[2\ 0\ 1]^T$, $\mathbf{v}_2=[1\ 2\ 0]^T$, $\mathbf{v}_3=[1\ 1\ 1]^T$ and $\mathbf{w}_1=[6\ 3\ 3]^T$, $\mathbf{w}_2=[4\ {-1}\ 3]^T$, $\mathbf{w}_3=[5\ 5\ 2]^T$. Find the *transition matrix* $\boldsymbol{P}$ from the T-basis to the S-basis. (5%)

9. Let $L{:}P_1\to P_2$ be defined by $L(p(x))=xp(x)$. Find the matrix of L with respect to the basis $S=\{x, 1\}$ and $T=\{x^2, x-1, x+1\}$ for P_1 and P_2, respectively. (5%)

10. Show that if matrix A is singular, then matrix **adj** A is singular. (5%)

11. Evaluate $A=\begin{vmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 3 \\ -1 & 2 & -3 & 4 \\ 0 & 5 & 0 & -2 \end{vmatrix}$ (5%)

12. Let A be a 2×2 matrix. If $tr(A)=7$ and $det(A)=12$, what are the eigenvalues of A. (5%)

13. An $n\times n$ matrix A is said to be *idempotent* if $A^2=A$. Show that if λ is an eigenvalue of an idempotent matrix, then λ must be either 0 or 1. (5%)

14. If A is an $n\times n$ matrix, then A is called ***nilpotent*** if $A^k=O_n$ for some positive integer k. (a) Show that every nilpotent matrix is singular. (b) If A is *nilpotent*, show that $I_n - A$ is nonsingular. (10%)

15. Find the *orthogonal* matrix P such that $P^{-1}AP=D$, a diagonal matrix. $A=\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$ (10%)