

中原大學 97 學年度碩士班入學考試

4 月 13 日 14:00~15:30 工業與系統工程學系(甲、乙組) 誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：機率與統計

(共 2 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

NOTE: Show only the equation if you need to use any statistical table (such as Z, t, or F) to obtain the value.

1. A normal population has mean μ and variance σ^2 . A sample with size n was randomly selected from the population with average \bar{X} and variance S^2 .

a.) Show that S^2 is an unbiased estimator of σ^2 (15 points)

b.) Find the maximum likelihood estimator of σ^2 (10 points)

2. Let X has a Poisson distribution with parameter λ . Show that the

moment-generating function of $Y = \frac{X - \lambda}{\sqrt{\lambda}}$ is given by $M_Y(t) = e^{(\lambda e^{\frac{t}{\sqrt{\lambda}} - 1} - \lambda)}$.

(10 points)

3. A researcher would like to know if the temperature and humility would affect student's attendance. The variables, X and Y , represent humility and temperature, respectively. Let (X, Y) have the joint p.f. given by the following table with the probability of absence:

$X \backslash Y$	0	1	2
1	1/4	0	1/4
3	1/12	1/3	1/12

a.) Are X and Y conditionally independent, given $Y \neq 1$? Why? (5 points)

b.) Are X and Y conditionally independent, given $Y \neq 2$? Why? (5 points)

4. There are four turning machines, A, B, C, and D, in a job shop factory. They were utilized to fabricate the same product H. The capacity of these machines is different. The Machine A is 25% faster than the Machine C to fabricate a product. The Machine B is 10% slower than the Machine D, and The Machine C is 5% faster than the Machine B. The percentage of defect for the Machine A, B, C, and D is 6%, 4%, 3% and 4%, respectively.

a.) What will be the percentage of defect for the product H? (10 points)

b.) If a new product L will share these machines with product H. For the product L, the percentage of defect for the Machine A, B, C, and D is 3%, 5%, 4%, 4%, respectively. If the ratio of fabricating product H and L is 60% to 40%. What will be the probability that a defect of Product L is fabricated by the Machine B? (10 points)

5. Statement: When the $cov(X, Y) = 0$, then the X and Y are independent.
 Is the statement true or false? If it is true, please prove it. Otherwise, please give one example to explain it. (10 points)
6. A professor would like to know if the selection of text book will affect students' learning performance. He/She used two different books, A and B , in two classes, X and Y , and randomly selected seven students from each class to do the pre-test and post-test. The pre-test has been conducted at the beginning of the semester and the post-test has been done in the final week of that semester. The following table shows the score from these tests. Please discuss and test if the text book will affect students' performance. Show all your works. (25 points)

Student #		1	2	3	4	5	6	7
Class X	Pre-test	57	44	63	52	32	47	52
	Post-test	78	69	82	69	80	84	72
Class Y	Pre-test	44	51	66	60	38	47	57
	Post-test	77	70	72	74	60	miss	62