中原大學 97 學年度碩士班入學考試

4 月 13 日 16:00~17:30 工業與系統工程學系乙組　　誠實是我們珍視的美德，我們喜愛「拒絕作弊，堅守正直」的你！

科目：作業研究　　　　　　　　　　　　　　　　（共 3 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　■不可使用計算機

(1) (18%) Investor Dow has $10,000 to invest in four projects. The following table gives the cash flow for the four investments.

Project	Year 1	Year 2	Year 3	Year 4	Year 5
1	-1.00	0.40	0.35	1.65	1.25
2	-1.00	0.60	0.25	1.52	1.32
3	0.00	-1.00	0.80	1.90	0.75
4	-1.00	0.42	0.63	1.78	0.96

The information in the table can be interpreted as follows: For project 1, $1.00 invested at the start of year 1 will yield $0.40 at the start of year 2, $0.35 at the start of year 3, $1.65 at the start of year 4, and $1.25 at the start of year 5. The remaining entries can be interpreted similarly. The entry 0.00 indicates that no transaction is taking place. Dow has the additional option of investing in a bank account that earns 6% annually. All funds accumulated at the end of one year can be reinvested in the following year.

Solution: Let x_i = Dollars invested in project i, i = 1,2,3,4

y_j = Dollars invested in bank in year j, j = 1,2,3,4

z = Dollars at the start of year 5.

Objective function : Max z

Continue to formulate the problem as a linear programming program to determine the optimal allocation of funds to investment opportunities.

(2) Consider the following LP primal model.

$$\begin{aligned} \text{Maximize } z = \ & 3X_1 + 2X_2 + 5X_3 \\ \text{s.t. } \ & X_1 + 2X_2 + X_3 \leq 430 \\ & 3X_1 + 2X_3 \leq 460 \\ & X_1 + 4X_2 \leq 420 \\ & X_j \geq 0 \text{ for all j.} \end{aligned}$$

Optimal solution: $X_1 = 0$, $X_2 = 100$, $X_3 = 230$, $z = 1350$.

(i) (5%) Construct the dual problem for this primal problem.

(ii) (10%)Use the **complementary slackness property** and the optimal solution for the primal problem to find the optimal solution for the primal problem. Show your calculation.

(3) Dynamic Programming Problem.

An electronic device consists of three components. The three components are in series so that the failure of one component causes the failure of the device. The reliability (probability of no failure) of the device can be improved by installing one or two standby units in each component. The following table charts the reliability, r, and the cost, c. The total capital available for the construction of the device is $10,000. How should the device be constructed? (*Note*: The objective is to maximize the reliability, $r_1r_2r_3$, of the device.)

No. of parallel units	Component 1		Component 2		Component 3	
	r_1	c_1 ($)	r_2	c_2 ($)	r_3	c_3 ($)
1	.6	1000	.7	3000	.5	2000
2	.8	2000	.8	5000	.7	4000
3	.9	3000	.9	6000	.9	5000

(i) (10%)Develop the backward recursive relationship equation.

(ii) (15%)Use the recursive relationship to find the optimum solution.

(4) Consider a self-service model in which the customer is also the server. Note that this corresponds to having an infinite number of servers available. Customers arrive according to a Poisson process with parameter λ, and service times have an exponential distribution with parameter μ.

(i) (5%) Find L_q and W_q.

(ii) (10%) Construct the rate diagram for this queueing system.

(iii) (5%) Find L and W.

(5) (12%) A store sells a special item whose daily demand can be described by the following pdf:

Daily demand, D	0	1	2	3
$P\{D\}$	.1	.3	.4	.2

The store is comparing two ordering policies: (1) Order up to 2 units every day if the stock level is less than 2, else do not order. (2) Order 1 unit every day if the stock level is less than 2, else do not order.

Express the two ordering policies as a Markov chain.

(6) (10%)You are given an M/M/1 queueing system in which the expected waiting time and expected number in the system are 120 minutes and 8 customers, respectively. Determine the probability that a customer's service time exceeds 20 minutes.

(Hint: $W = \frac{1}{\mu - \lambda}$ and P(T_S >A) = $e^{-\mu A}$, where T_s denotes the customer service time)