

# 中原大學 97 學年度碩士班入學考試

4 月 13 日 11:00~12:30 應用數學系統計組

誠實是我們珍視的美德，  
我們喜愛「拒絕作弊，堅守正直」的你！

科目：基本數學(含微積分及線性代數)

(共 1 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

每題 10 分，共 10 題。

1. Use the logarithmic differentiation to find  $f'(0)$  where

$$f(x) = \frac{(x^3 + 2)^4 e^x \cos^3 x}{\sqrt[3]{x-1}(x+1)^{\ln(x+1)}}.$$

2. Evaluate (a)  $\lim_{x \rightarrow 0^+} x \ln x$  (b)  $\int_0^1 \ln x dx$ .

3. Find the interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{4^n (n+1)}$ .

4. Locate all relative extrema and saddle points of the function

$$f(x, y) = xy - x^3 - y^2.$$

5. Evaluate the integral by first reversing the order of integration  $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dx dy$ .

6. Evaluate the determinant  $\det A$  of the  $n \times n$  real matrix

$$A = \begin{pmatrix} 1+a & a & \cdots & a \\ a & 1+a & \cdots & a \\ \vdots & \vdots & \ddots & \vdots \\ a & a & \cdots & 1+a \end{pmatrix}_{n \times n}.$$

7. Let  $T: R^2 \rightarrow R^3$  be a linear transformation such that  $T(2,1) = (3,1,9)$  and  $T(3,2) = (5,2,0)$ . Find  $T(x, y)$ .

8. Apply Gram-Schmidt process to the subset  $\{(1,0,1), (0,1,1), (1,1,0)\}$  of  $R^3$  with the standard inner product, then find the corresponding orthonormal set.

9. Use Cayley-Hamilton theorem to compute  $A^5 - 5A^3 + 3A + 2I$  where

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}.$$

10. Find a matrix  $Q$  such that  $Q^{-1}AQ$  is a diagonal matrix where

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}.$$