

元智大學 九十七 學年度研究所 碩士班 招生試題卷

系(所)別： 電機工程學系碩  
士班

組別： 控制工程組

科目： 工程數學

用紙第 / 頁共 / 頁

●不可使用電子計算機

1. (10%) Find the Laplace transform of  $f(t) = \begin{cases} \cos 2t, & 0 \leq t < 2\pi \\ 0, & t \geq 2\pi \end{cases}$ .

2. (15%) Use convolution method to evaluate the inverse Laplace transform of  $\frac{2}{(s-1)(s^2+4)}$ .

3. (10%) Evaluate the following integral  $\oint_C \frac{\sin z}{z^2(z^2+4)} dz$ , where  $C$  is a simple closed path enclosing both  $0$  and  $2i$ , but not  $-2i$ .

4. (15%) Use Laplace transform to solve the initial value problem

$$y'(t) - 3y(t) = 3u(t-4), \quad y(0) = 2$$

where  $u(t)$  is the unit step function defined by  $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$ .

5. (14%) Let the function  $f(x)$  be defined on the interval  $-L \leq x \leq L$ . Show the Fourier coefficients  $a_n$  and  $b_n$  in the Fourier series representation of

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}$$
 are given by the formulas

$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx; \quad a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, \text{ for}$$

$$n = 1, 2, \dots$$

6. Let  $f(x) = \begin{cases} -1, & -4 \leq x < 0 \\ 0, & x = 0 \\ 1, & 0 < x \leq 4 \end{cases}$

6.1. (12%) Find the Fourier series of  $f(x)$ .

6.2. (6%) Discuss the convergence of the Fourier series in 6.1.

6.3. (6%) What is Gibbs phenomenon? Do you expect to see Gibbs phenomenon of the Fourier series of  $f(x)$ ?

7. (12%) Write out the definition of the following.

- |  |                                   |
|--|-----------------------------------|
| (a), Covariance of two random variables, | (b), Orthogonal random variables, |
| (c), Uncorrelated random variables,      | (d), Gaussian random variable.    |