●不可使用電子計算機

1. (15%) Use Laplace transform to solve the initial value problem

$$y'(t)-3y(t)=3u(t-4),\quad y(0)=2$$

where $u(t)$ is the unit step function defined by $u(t)=\begin{cases}1, & t\ge 0\\ 0, & t<0\end{cases}$.

2.1. (5%) State the convolution theorem for Laplace transform.

2.2. (5%) Find the convolution $t*e^t$ of t with e^t for $t\ge 0$.

2.3. (5%) Find the Laplace transform of $t*e^t$, and with it to verify the convolution theorem.

3. (10%) Evaluate the following integral $\oint_C \frac{1}{z^2+1}dz$, where C is a simple closed path enclosing both i and $-i$. Show your detailed reasonings.

4. (10%) Use residue theorem to evaluate $\oint_C \frac{30z^2-23z+5}{(2z-1)^2(3z-1)}dz$, $C:\ |z|=1$.

5. (12%) Let the function $f(x)$ be defined on the interval $-L\le x\le L$. Show the Fourier coefficients a_n and b_n in the Fourier series representation of

$f(x)\sim a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L}$ are given by the formulas

$a_0=\frac{1}{2L}\int_L^L f(x)dx;\ a_n=\frac{1}{L}\int_L^L f(x)\cos\frac{n\pi x}{L}dx,\ b_n=\frac{1}{L}\int_L^L f(x)\sin\frac{n\pi x}{L}dx$, for

$n=1,2,\cdots$

6. Let $f(x)=\begin{cases}-\pi/4, & -\pi\le x<0\\ 0, & x=0\\ \pi/4, & 0<x\le\pi\end{cases}$

6.1. (12%) Find the Fourier series of $f(x)$.

6.2. (8%) Discuss the convergence of the Fourier series in 6.1.

6.3. (6%) What is Gibbs phenomenon? Do you expect to see Gibbs phenomenon of the Fourier series of $f(x)$?

7. (12%) The joint probability density function (PDF) of X and Y is

$f_{X,Y}(x,y)=\begin{cases}5y/4, & -1\le x\le 1,\ x^2\le y\le 1\\ 0, & \textit{otherwise}\end{cases}$. Find the marginal PDFs of $f_X(x)$ and $f_Y(y)$.