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1. (Differential Equation **15%**)
   Solve the initial value problem: $y''+4y'+3y = e^{-x}+e^{-3x}$ $y(0)=2, y'(0)=1$

2. (Laplace Transforms **25%**)
   (a). (**5%**) Find the Laplace transform of the function, $f(t)=t$.
   (b). (**10%**) Given $J_0(t)=\sum_{m=0}^{\infty}\frac{(-1)^m t^{2m}}{2^{2m}(m!)^2}$ and the Laplace transform of $\frac{t^{n-1}}{(n-1)!}$ is $\frac{1}{s^n}$, where $n=1,2,\cdots$. Prove that the Laplace transform of the $J_0(2\sqrt{kt})$ is $\frac{1}{s}e^{-k/s}$, where k is a constant.
   (c). (**10%**) Use the Laplace transforms to solve the following integral equation: $y(t)+2e^{t}\int_0^{t} e^{-\tau}y(\tau)d\tau = te^{t}$

3. (Vector Analysis **10%**)
   Let $f(x,y,z)=zy+yx$ and $\bar{v}=[v_x(x,y,z),v_y(x,y,z),v_z(x,y,z)]=[y^2,z^2,x^2]$. Find the following:
   (a). (**5%**) The gradient of f and the curl of the gradient of f.
   (b). (**5%**) The curl of $\bar{v}$ and the divergence of the curl of $\bar{v}$.
   (c).

4. (Fourier Analysis **30%**)
   The Fourier transform of a function, *f(t)*, is defined by $\Im[f(t)]=F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$.
   (a). (**10%**) Find the Fourier series of the function $f(t)$, of period $p=2$.

   $$f(t)=1-t^2,\quad (-1<t<1)$$

   (b). (**10%**) The convolution $f*g$ of functions f and g is defined by $(f*g)(t)=\int_{-\infty}^{\infty}f(t-\tau)g(\tau)d\tau$. Prove that $\Im[f*g]=\sqrt{2\pi}\,\Im[f(t)]\Im[g(t)]$.
   (c). (**10%**) Find the Fourier transform of the following function:

   $$f(t)=\begin{cases}-1 & if\quad -1<t<0\\ 1 & if\quad 0<t<1\\ 0 & otherwise\end{cases}$$

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5. (Linear Algebra **10%**)
Find the eigenvalues and their corresponding eigenvectors of

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

6. (Partial Differential Equations **10%**)
The one-dimensional wave equation is given by

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

where $u(x,t)$ and c is a constant. The boundary and initial conditions are listed below:

(Space) Boundary condition $u(x,t)\big|_{x=0}=0 \text{ and } u(x,t)\big|_{x=L}=0$

(Time) Initial condition $u(x,t)\big|_{t=0}=0 \text{ and } \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=0.$

Please solve this one-dimensional wave equation.