元智大學 九十七 學年度研究所 **碩士班** 招生試題卷

系(所)別：工業工程與管理學系碩士班　組別：不分組　科目：微積分　用紙第 1 頁共 1 頁

●不可使用電子計算機

1. (10%) An open box is constructed from cardboard by cutting out squares of equal size in the corners and then folding up the sides. If the cardboard is 5 inches by 10 inches, determine the volume of the largest box which can be so constructed.

2. (10%) Find the solution to the initial value problem $e^{x^2}y' = xe^{y}$ with the initial condition $y(0) = 0$.

3. (10%) Evaluate the indefinite integral $\int \cos(\ln x)dx$.

4. (10%) Find the indefinite integral $\int \frac{7+11(\ln x)^2}{x(\ln x)^3 + x\ln x}dx$.

5. (20%) Let $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ and $g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$.

   a. Find the intervals of convergence of f and g. (5%)
   b. Compare $f'(x)$ with $g(x)$. (5%)
   c. Compare $\int f(x)dx$ with $g(x)$. (5%)
   d. Identify the functions f and g. (5%)

6. (10%) Find the equation of the line in the plane $x = 1$ that is tangent to the curve of intersection of this plane with $z = 4 - y^2 - x^2$ at (1, 1, 2).

7. (10%) Determine the maximum revenue obtained by a furniture store that sells two competitive products, the prices of which are p_1 and p_2 if the total revenue is given by $R = 300p_1 + 900p_2 + 1.8p_1p_2 - 1.5p_1^2 - p_2^2$.

8. (10%) Use spherical coordinates to find the mass of the conical solid bounded by the graphs of $z = \sqrt{x^2 + y^2}$ and $z = 4$ if the density is $k\sqrt{x^2 + y^2}$.

9. (10%) Let R be the region bounded by the graphs of $x + y = 1, x + y = 2, 2x - 3y = 2,$ and $2x - 3y = 5$. Use the change of variables, $x = \frac{1}{5}(3u + v)$, $y = \frac{1}{5}(2u - v)$ to evaluate the integral $\iint_R (2x - 3y)dA$.