

系別：機械與機電工程學系

科目：工程數學

准帶項目請打「V」

簡單型計算機

本試題共 5 頁，5 大題

1. (15%) Solve $xy' = \frac{y^2}{x} + y$.

2. (20%) Find the general solution of the differential equation.
 $y'' - 3y' + 2y = 10\sin(x)$

3. (20%) Use the Laplace transform to solve the initial value problem.
 $y'' + 2y' + 2y = \delta(t - 4); y(0) = y'(0) = 0$

4. (20%) Solve the partial differential problem.

$$\frac{\partial u^2}{\partial x^2} + \frac{\partial u^2}{\partial y^2} = 0 \quad (x > 0, y < 0),$$

$$u(x, 0) = 0 \quad (x > 0),$$

$$u(0, y) = \begin{cases} 0, & \begin{cases} -5 \leq y \leq 0 \\ y < -7 \end{cases} \\ 2, & -7 \leq y < -5 \end{cases}$$

5. (25%) Evaluate both sides of Stokes's Theorem,

$$\oint_C \vec{v} \cdot d\vec{R} = \int_S \vec{n} \cdot \nabla \times \vec{v} \, d\sigma,$$

where $\vec{v} = 2\vec{i} + yz^2\vec{j} + x\vec{k}$ and S is the plane with corners at $(1,0,0)$, $(0,2,0)$, $(0,0,1)$ as shown, and verify that the results do agree.

