

淡江大學 97 學年度碩士班招生考試試題

系別：數學學系

A

科目：代 數 學

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- Find the remainder when:
(a) 10^{516} is divided by 7
(b) 481^{5251} is divided by 53 (10%)
- Let G be the rational numbers except -1 .
Show that $(G, *)$ is a group where $a * b = a + b + ab$ for all $a, b \in G$. (10%)
- In each case determine all subgroups of G and draw the lattice diagram.
(a) $G = Z_{45}$ (b) $G = Z_{18}^*$, where $Z_{18}^* = \{a \in Z_{18} \mid a \text{ and } 18 \text{ are relatively prime}\}$ (c) $G = S_3$ (20%)
- Prove that every finite integral domain is a field. (10%)
- Let G be a group and
 $(ab)^3 = a^3b^3$
 $(ab)^4 = a^4b^4$
 $(ab)^5 = a^5b^5$
for all a and b in G .
Show that G is abelian. (10%)
- Show that a group G is abelian if $x^2 = e$ for all x in G . Give an example showing that the converse is false. (10%)
- Find all groups of order 77 up to isomorphism. (10%)
- Find all groups of order 55 up to isomorphism. (10%)
- Find the Galois group of $x^2 - 2$ over \mathbb{Q} . (10%)