淡江大學97學年度碩士班招生考試試題

系別：數學學系　　　A　　　科目：線　性　代　數

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#務必書寫過計算程，否則不予計分。

1. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & -6 \\ 0 & 0 & 2 \end{bmatrix}$.

   (a) Find the characteristic polynomial of A. (5points)

   (b) Find an invertible matrix P such that $P^{-1}AP$ is diagonal. (10 points)

   (c) Find A^8. (5points)

2. Let A be m×n matrix. Show that the column space of A and the null space of A^T are orthogonal. (10 points)

3. Let B= {(1,1,0), (1,2,0), (0,1,2)} and $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

   (a) Show that A is invertible and find A^{-1}. (10 points)

   (b) Show that B is a basis for R^3. (5 points)

   © Let R^3 be the inner product space with the Euclidean inner product. Use the Gram-Schmidt orthonormalization process to transform the basis B into an orthonormal basis. (10 points)

   (d) Let W = span{(1,1,0), (1,2,0)}. Find the orthogonal projection of (1,-1,2) onto W. (5points)

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4. Let P_1 be the set of all polynomials of degree at most 1 and B={1, x}. Let T : $R^3 \to P_1$ be defined by T(a,b,c)=2c-b+(a-b)x and D={(1,0,0),(0,1,0),(0,1,1)}.

(a) Find the kernel of T. (5 points)

(b) Find the matrix of T corresponding to the ordered bases D and B. (10 points)

5. Let $A = \begin{bmatrix} 1 & -1 & 2 & -2 & 3 \\ 2 & -2 & 4 & -4 & 6 \\ 0 & 0 & 0 & 0 & 1 \\ 4 & -5 & 7 & -7 & 11 \end{bmatrix}$, $b=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ and $X=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$.

(a) Find a necessary and sufficient condition on b such that AX=b is consistent and find the general solutions of AX=b. (10 points)

(b) Find Rank(A). (5points)

6. Let M be the vector space of all 2×2 matrices. Let $A=\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ and

U={X $\in$ M |AX=XA}.

(1) Show that U is a subspace of M. (5points)

(2) Find the dimension of U. (5points)