

# 淡江大學 97 學年度碩士班招生考試試題

系別：數學學系

科目：統 計 學

A

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁，5 大題

1. (15 points) A random sample  $X_1, X_2, \dots, X_n$  from a normal distribution with  $\sigma = 1$  is to be used to test the null hypothesis  $\mu = \mu_0$  against the alternative hypothesis  $\mu = \mu_1$ , where  $\mu_1 > \mu_0$ . Use the Neyman-Pearson lemma to find the best critical region of size  $\alpha$ .

2. (15 points) Let  $X_1, X_2, X_3$  have a continuous joint distribution with a density

$$f(x_1, x_2, x_3) = \begin{cases} q(ax_1 + bx_2 + cx_3) & \text{for } 0 \leq x_i \leq 1, i = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

where  $a, b, c$  are positive constants. Find:

- (a)  $q$  as a function of  $(a, b, c)$ .
- (b) The conditional density of  $(X_1, X_2)$  given  $X_3 = x_3$ .
- (c) The conditional density of  $X_3$  given  $X_1 = x_1, X_2 = x_2$ .

3. (20 points) Is the statistic  $Y = (X_1 + 2X_2 + 3X_3)/6$  sufficient for estimating the parameter  $\theta$  of a Bernoulli population?

4. (20 points) Let  $Y_{(1)} < Y_{(2)}$  denote the order statistics of a random sample of size 2 from a normal distribution with mean 0 and variance  $\sigma^2$ . Find  $E(Y_{(1)})$ .

5. (30 points) Let  $\alpha, \beta, \gamma$  be arbitrary positive constants and let  $U$  and  $V$  be independent chi-square random variables with  $m$  and  $n$  degrees of freedom, respectively. Let  $Z = \gamma U/V$  and  $W = \alpha U + \beta V$ .

- (a) Find the conditional probability density function of  $W$  given  $Z = z$ .
- (b) Find the distribution of  $T$  where  $T$  is defined as

$$T = \frac{(\gamma + Z)W}{\alpha Z + \beta \gamma}$$