

淡江大學 97 學年度碩士班招生考試試題

系別：物理學系

科目：物理數學

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1. Evaluate the integral $I = \iiint \ln(x^2 + y^2) dx dy dz$ over the interior of the conical region $x^2 + y^2 \leq z^2, 0 \leq z \leq 1$. (15 points)

2. Given $A = \begin{pmatrix} -3 & 2 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, and $f(A) = A^3 - 2A^2 + 5A - 3$, find $f(A)$. (15 points)

Note: Evaluating each term in $f(A)$ and then adding it up are not credited.

3. (a) List mathematical quantities which are invariant under similarity transformation. (5 points)

(b) Write down the definition of Levi-Civita symbol ε_{ijk} and show that $\sum_{i,j,k=1}^3 \varepsilon_{ijk} = 0$. (10 points)

4. A matrix is both Hermitian and Unitary. Show that its eigenvalues are ± 1 . (15 points)

5. Suppose that a particle in one dimensional system is at position $x = x_0$, show that momentum uncertainty of the particle is infinite. (10 points)

6. Solve the following ordinary differential equation. (10 points)

$$-3xy^2 y' + (2x^3 + y^3) = 0$$

7. Solve the following partial differential equation. (20 points)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x, \text{ where } 0 \leq x \leq 1 \text{ and } t > 0.$$

$$\text{BC: } u(0,t) = u(1,t) = 0, \text{ IC: } u(x,0) = \frac{x^3}{6}, \frac{\partial u(x,0)}{\partial t} = 0.$$