請依題號順序作答或留白，數值答案取小數點後兩位作答(第三位四捨五入)
共兩大題，除第一大題第一小題為 20 分外，其餘每小題 10 分，合計 100 分。

I.

1. The following table gives the joint probability distribution, $f(X,Y)$, of two random variables X and Y.

		X		
		1	2	3
Y	1	0.03	0.06	0.06
	2	0.02	0.04	0.04
	3	0.09	0.18	0.18
	4	0.06	0.12	0.12

(a) Find the marginal distribution of X, namely, $f(X)$.
(b) Find the conditional PDF, $f(X \mid Y)$.
(c) Are X and Y independent variables? How do you know?
(d) Find the conditional expectation E(Y|X).

2. In actuarial science, one of the models used for describing mortality is

$$f(t) = kt^2(100-t)^2 \qquad 0 \le t \le 100$$

where t is the age at which a person dies.
(a) Let A be the event "Person lives no more than 60". Find the probability of A, P(A).
(b) What is the probability that a person will die between the ages of 65 and 85 given that that person has lived to be at least 60?

3. Consider the following two cases:
(a) Two fair dice (one red and one blue) are thrown, let A be the event the red die shows a 3, a 4, or a 5; let B be the event the blue die shows a 1 or a 2; and let C be the event the dice total is 7.
(b) Two fair dice (one red and one blue) are thrown, let A be the event the red die shows a 1, or a 2; let B be the event the blue die shows a 3, a 4, or a 5; and let C be the event the dice total is 4, 11, or 12.

Which of the following are correct?
(1) $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ is satisfied in case (a) and in case (b).

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系別：保險學系　　　　科目：統　計　學

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(2) $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ is satisfied in case (a) but not in case (b).

(3) $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ is satisfied in case (b) but not in case (a).

(4) $P(A \cap B \cap C) \neq P(A) \cdot P(B) \cdot P(C)$ for both cases (a) and (b).

(5) A, B, and C are independent in case (a) and in case (b).

(6) A, B, and C are independent in case (a) but not in case (b).

(7) A, B, and C are independent in case (b) but not in case (a).

(8) A, B, and C are not independent in case (a), and A, B, and C are not independent in case (b).

4. Urn I contains four white chips and five red chips; urn II has five white chips and four red chips. Three chips are drawn at random from urn I and transferred to urn II. Then a single chip is to be drawn from urn II. What is the probability that the chip drawn from the second urn will be red?

II.

5. Consider the following game. A fair coin is flipped until the first head appears; we win \$ $\log 2$ if it appears on the first toss, \$ $\log 2^2$ if it appears on the second toss, and, in general, \$ $\log 2^k$ if it first occurs on the kth toss. Let the random variable X denote our winnings. How much should we have to pay in order for this to be a fair game? [**Note**: A fair game is one where the difference between the ante and E(X) is zero.]

6. Let $X_1, \ldots, X_5$ be a random sample from a normal distribution with unknown mean and known variance equal to 16. Let $S^2 = \dfrac{\sum (X_i - \overline{X})^2}{4}$. Find M, if $P(S^2 \le M) = 0.05$.

7. Let X be a random variable with finite variance. Find $Corr[(X+Y)X, X]$ if Y = 12 – X. (**Note**: The correlation coefficient of two random variables X and Y is denoted by $\rho_{XY} = Corr(X,Y)$)

8. Let $X_1, \ldots, X_{12}$ be a random sample from a normal distribution with mean μ and variance σ^2. Find the value of k such that $\bar{x} \pm k\sqrt{\sum_{i=1}^{12}(x_i - \bar{x})^2}$ are the endpoints for a 90% confidence interval for μ.

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9. Let $X \sim N(\mu,\sigma^2)$. A random sample of three observations was obtained from this population. Consider the following estimators of μ:

$$\hat{\mu}_1=\frac{X_1}{2}+\frac{X_2}{3}+\frac{X_3}{6},\quad \hat{\mu}_2=\frac{X_1}{2}+\frac{X_2}{4}+\frac{X_3}{4},\ \text{and}\ \hat{\mu}_3=\frac{5X_1}{12}+\frac{5X_2}{12}+\frac{X_3}{6}$$

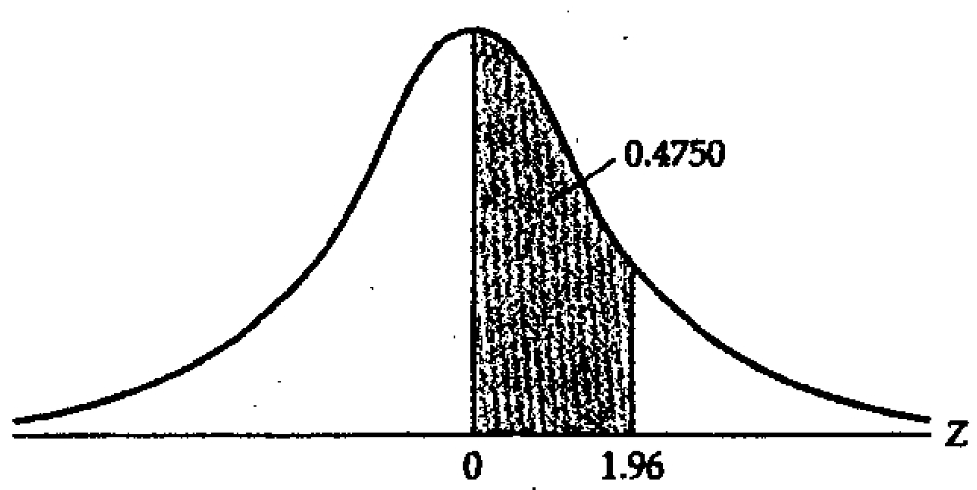
Which one would you choose? Why?

TABLE A-1a AREAS UNDER THE STANDARDIZED NORMAL DISTRIBUTION.

Example

Pr(0 ≤ Z ≤ 1.96) = 0.4750

Pr(Z ≥ 1.96) = 0.5 − 0.4750 = 0.025



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4454	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: This table gives the area in the right-hand tail of the distribution (i.e., $Z \geq 0$). But since the normal distribution is symmetrical about $Z = 0$, the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example, $P(-1.96 \leq Z \leq 0) = 0.4750$. Therefore, $P(-1.96 \leq Z \leq 1.96) = 2(0.4750) = 0.95$.

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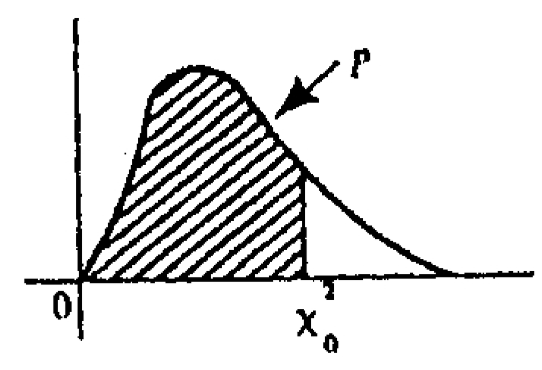
The table below gives the value χ_0^2 for which $P(\chi^2 < \chi_0^2) = P$ for a given number of degrees of freedom and a given value of P.

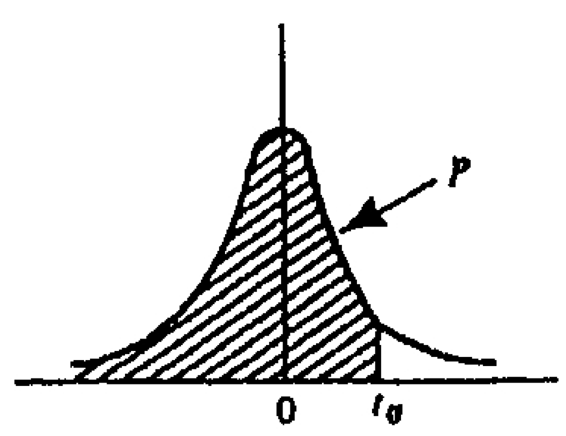
χ^2 Distribution



Degrees of freedom	Value of P							
	0.005	0.010	0.025	0.050	0.950	0.975	0.990	0.995
1	0.00	0.00	0.00	0.00	3.84	5.02	6.64	7.88
2	0.01	0.02	0.05	0.10	5.99	7.38	9.21	10.60
3	0.07	0.12	0.22	0.35	7.82	9.35	11.35	12.84
4	0.21	0.30	0.48	0.71	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.96
9	1.74	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.58	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.08	4.66	5.63	6.57	23.69	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.27	7.02	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00

The table below gives the value t_0 for which $P(-\infty < t < t_0) = P$ for a given number of degrees of freedom and a given value of P.

Student's *t* Distribution



Degrees of freedom	Value of P				
	0.900	0.950	0.975	0.990	0.995
1	3.08	6.31	12.71	31.82	63.66
2	1.89	2.92	4.30	6.97	9.93
3	1.64	2.35	3.18	4.54	5.84
4	1.53	2.13	2.78	3.75	4.60
5	1.48	2.02	2.57	3.37	4.03
6	1.44	1.94	2.45	3.14	3.71
7	1.42	1.90	2.37	3.00	3.50
8	1.40	1.86	2.31	2.90	3.36
9	1.38	1.83	2.26	2.82	3.25
10	1.37	1.81	2.23	2.76	3.17
11	1.36	1.80	2.20	2.72	3.11
12	1.36	1.78	2.18	2.68	3.06
13	1.35	1.77	2.16	2.65	3.01
14	1.35	1.76	2.15	2.62	2.98
15	1.34	1.75	2.13	2.60	2.95
20	1.33	1.73	2.09	2.53	2.85
25	1.32	1.71	2.06	2.49	2.79
30	1.31	1.70	2.04	2.46	2.75
60	1.30	1.67	2.00	2.39	2.66
∞	1.28	1.65	1.96	2.33	2.58