國立高雄大學九十七學年度研究所碩士班招生考試試題

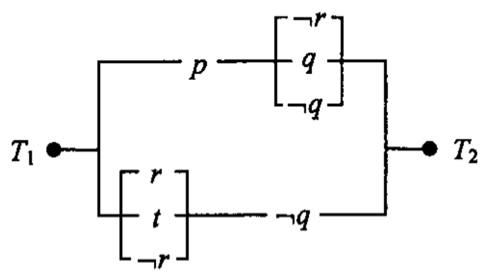
科目：離散數學
考試時間：100 分鐘

系所：
資訊工程學系碩士班
本科原始成績：100 分

是否使用計算機：否

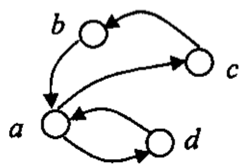
1. True/False (3% for each; total 30%)
   (1) Let $m, n \in Z^+$. Then $n \cdot C(m+n, m) = (m+1) \cdot C(m+n, m+1)$
   (2) + 5 is a positive integer.? is a statement.
   (3) $\forall x\ \exists y\ [p(x, y) \rightarrow q(x, y)] \Leftrightarrow \exists y\ \forall x\ [\neg p(x, y) \vee q(x, y)]$
   (4) For sets $A, B, C \subseteq \mathrm{U}$. If $A \subseteq B$ and $B \not\subset C$, then $A \not\subset C$.
   (5) Let $m, n \in Z^+$. If m, n are perfect squares, then $m + n$ is a perfect square.
   (6) Let f: $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{Z}$ be defined by $f(a, b) = \lceil a + b \rceil$. Then f is associative.
   (7) Let f, g: $\mathbf{Z}^+ \rightarrow \mathbf{Z}^+$ where for all $x \in \mathbf{Z}^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x-1\}$. Then f is a one-to-one and onto function while g is onto but not one-to-one.
   (8) For an alphabet Σ, let $A, B, C \in \Sigma^*$. Then $A(B \cap C) \subseteq AB \cap AC$.
   (9) A relation is called a partial order if it is reflexive, symmetric and transitive.
   (10) If $a \mid (bx + cy)$, then $a \mid b$ or $a \mid c$, $\forall a, b, c \in \mathbf{Z}$.

2. Simplify the following network. (10%)



3. Five speakers (A, B, C, D, and E) are scheduled to present papers in a conference.
   (a) How many ways can this be arranged? (5%)
   (b) How many ways can this be arranged without B speaking before A? (5%)
   (b) How many ways can this be arranged if A speaks immediately before B? (5%)

4. Consider the following directed graph.



   (a) Find the corresponding adjacency matrix. (5%)
   (b) Find the transitive closure of the relation represented by this graph. (5%)
   (c) Identity what properties the relation corresponding to this graph satisfy. Reflexivity?

Irreflexivity? Symmetry? Antisymmetry? Asymmetry? (8%)

(d) Does this graph contain an Euler cycle? If yes, show the cycle. If not, explain why? (7%)

5. Let input and output alphabet $I = O = \{a, b\}$. Construct a state diagram for a finite state machine that can recognize $(aa^*bb^*)^*$. (10%)

6. Two integers a and b are **congruent modulo *n*** if $n \mid (a - b)$, denoted as $a \equiv b \pmod{n}$. Show that congruence modulo n is an equivalence relation on $\mathbf{Z}$. (10%)