

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：工程數學

考試時間：100 分鐘

系所：

土木與環境工程學系碩士班土木工程組 是否使用計算機：是  
本科原始成績：100 分

1. Solve the logistic differential equation  $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$ , where  $k$  and  $L$  are positive constants.

(20)

2. Solve the boundary value problem  $\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$  ( $0 \leq z \leq 2H$ ,  $t \geq 0$ ) with the following

boundary conditions:

$$u(z, 0) = u_0$$

$$u(0, t) = 0 \text{ and } u(2H, t) = 0 \text{ if } t > 0$$

where  $c_v$ ,  $H$  and  $u_0$  are positive constants. (30)

3. (a) Show how to find a particular solution by variation of parameters (15).

Consider a 2<sup>nd</sup> Order linear non-homogeneous ODE in (1)

$$y'' + p(x)y' + q(x)y = r(x) \quad (1)$$

One may find two basis functions to form the general solution for the ODE.

$$y_h = c_1 y_1 + c_2 y_2 \quad (c_1, c_2 = \text{const}) \quad (2)$$

And obtain the particular solution  $y_p$  of (1) in the form

$$y_p(x) = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx \quad (3)$$

where  $W = y_1 y_2' - y_2 y_1'$ .

- (b) Use (a) to find the complete solution to the ODE,  $y'' + 2y' + 2y = 2e^x \cos x$  (15).

4. Consider a system of two tanks as shown below. Find the salt content for each tank if the system can be modeled as

$$y_1'' = 4y_2 - 4e^t, y_2'' = 3y_1 + y_2, y_1(0) = 1, y_1'(0) = 2, y_2(0) = 2, y_2'(0) = 3$$

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