

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：線性代數(乙)  
考試時間：100 分鐘

系所：應用數學系碩士班乙組  
本科原始成績：100 分

是否使用計算機：是

Notations.

$I_n$  : the identity matrix of size  $n$ .  
 $M_{n \times m}(\mathbb{R})$ : set of  $n \times m$  real matrices.  
 $A^T$ : the transpose of matrix  $A$ .

- 1 a. (10) Let  $W_1$  and  $W_2$  be subspaces of a vector space  $V$ . Prove that the intersection  $W_1 \cap W_2$  is a subspace of  $V$ .  
b. (8) Let  $W_1 = \text{span}\{(1, 2, 3), (2, 1, 1)\}$  and  $W_2 = \text{span}\{(1, 0, 1), (3, 0, -1)\}$ . Find a basis for  $W_1 \cap W_2$ .
- 2 Let  $A \in M_{3 \times 3}(\mathbb{R})$  and  $A^2 - 3A + 2I_2 = 0$  where  $0$  is a zero matrix.  
a. (10) Show that  $A$  is diagonalizable.  
b. (8) Describe all matrices  $A$ .

- 3 a. (10) Let  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ . Find an orthogonal matrix  $Q$  and an upper triangular matrix  $R$  such that  $A = QR$ .  
b. (8) Let  $A \in M_{n \times n}(\mathbb{R})$  and  $A = QR$  where  $Q$  is orthogonal and  $R$  is upper triangular. Show that  $B = RQ$  is similar to  $A$ .

- 4 Let  $A = \begin{bmatrix} -1 & 3 & -3 \\ 0 & 5 & -6 \\ 0 & 3 & -4 \end{bmatrix}$  and  $\mathbf{x}_0 = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$ .  
a. (10) Find the eigenvalues and the corresponding eigenvectors of  $A$ .  
b. (5) Find the general solution  $\mathbf{x}_k$  of  $\mathbf{x}_k = A\mathbf{x}_{k-1}$  starting from  $\mathbf{x}_0$ .  
c. (8) Let  $\mathbf{B} = \{\mathbf{x} \mid \|\mathbf{x}\|_2 = 1\}$  and let  $F: \mathbf{B} \rightarrow \mathbf{B}$  be defined by  $F(\mathbf{x}) = \frac{A\mathbf{x}}{\|A\mathbf{x}\|}$ . Find  $\lim_{n \rightarrow \infty} F^n(\mathbf{x}_0)$ .
- 5 Let  $\mathbf{x} \in \mathbb{R}^n$  and  $A = I_n - \mathbf{x}\mathbf{x}^T$ .  
a. (8) Find the eigenvalues and the corresponding eigenspaces of  $A$ .  
b. (5) Show that  $\det(A) = 1 - \mathbf{x}^T \mathbf{x}$ .  
c. (10) Find  $A^{-1}$  if  $\mathbf{x}^T \mathbf{x} \neq 1$ .