

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：基礎數學
考試時間：100 分鐘

系所：統計學研究所碩士班
本科原始成績：100 分

是否使用計算機：是

1. (12%) Determine whether or not the vectors w_1 and w_2 , respectively, are in the vector space spanned by v_1 and v_2 where

$$v_1 = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 0 \end{bmatrix}, w_1 = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 5 \end{bmatrix}, w_2 = \begin{bmatrix} 4 \\ 0 \\ -4 \\ 5 \end{bmatrix}$$

2. (10%) If A is a positive definite $k \times k$ matrix, show that $a_{ii}a_{jj} > a_{ij}^2$ for all $i \neq j = 1, \dots, k$.

3. (10%) Let $B = \begin{bmatrix} -I & A - I \\ A - I & A \end{bmatrix}$, where A is an $n \times n$ symmetric idempotent matrix and I is the $n \times n$ identity matrix. Find the determinant of B .

4. (10%) Let A be an $n \times m$ matrix and B an $m \times k$ matrix. Show that the column space of AB is a subspace of the column space of A .

5. (23%) Evaluate the following integrals

(a) (7%) $\int_1^e x^2 \ln x dx$; (b) (8%) $\int_0^{1/2} x^4 / \sqrt{1 - x^2} dx$; (c) (8%) $\int_0^1 \int_0^{\sqrt{y}} y / \sqrt{4 - x^2} dx dy$

6. (10%) Show that $\lim_{x \rightarrow 0} (\sin x + \cos x - 1) / x = 1$.

7. (10%) Determine whether $\int_0^\infty [\ln(1 + x)] / (\sqrt{x} + x^2) dx$ converges or diverges. (You should give reasons for your answer)

8. (15%) Let $f_1(x) = x \exp\{-x^2\}$ and $f_2(x) = x / (1 + x^2)$. Clearly, both of them are even functions so that for any constant $a < \infty$, we have $\int_{-a}^a f_j(x) dx = 0$ ($j = 1, 2$). However, $\int_{-\infty}^\infty f_1(x) dx = 0$ but $\int_{-\infty}^\infty f_2(x) dx$ does not exist. Give your reasons.