

國立中央大學97學年度碩士班考試入學試題卷

所別：企業管理學系碩士班 一般類組(甲組) 科目：微積分 共 / 頁 第 / 頁

\*請在試卷答案卷(卡)內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find  $f(4)$  if  $\int_0^{x^2} f(t) dt = x \cos \pi x$ . Answer : \_\_\_\_\_

2. Integrate  $f(x, y, z) = \sqrt{x^2 + z^2}$  over the circle  $r(t) = (a \cos t)i + (a \sin t)k$ ,  $0 \leq t \leq 2\pi$ . Answer : \_\_\_\_\_

3. Find the value of  $a$  for which the limit  $\lim_{x \rightarrow 0} \frac{\sin(ax) - \sin x - x}{x^3}$  is finite. Answer : \_\_\_\_\_

4. Evaluate  $\iint_R e^{x^2+y^2} dy dx$ , where  $R$  is the semicircular region bounded by  $x$ -axis and the curve  $y = \sqrt{1-x^2}$ . Answer: \_\_\_\_\_

5. Find all values of  $x$  for which  $\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$  converges absolutely. Answer : \_\_\_\_\_

6. If  $x$  thousand dollars is spent on labor and  $y$  thousand dollars is spent on equipment, the output at a certain factory will be  $Q(x, y) = 60x^{1/3}y^{2/3}$  units. If \$120,000 is available, how should this be allocated between labor and equipment to generate the largest possible output? Answer : \_\_\_\_\_

7. Use the Lagrange multiplier  $\lambda$  to estimate the change in the maximum output of the factory in Problem 6. that will result if the money available for labor and equipment is increased by \$1,000. Answer : \_\_\_\_\_

8. Suppose we have a production function  $f(x, y) = 100x^{1/4}y^{3/4}$  where  $x$  is the number of units of labor,  $y$  is the number of units of capital, and  $f$  is the number of units of a certain product that is produced. Using the tangent plane, find an approximation for the change in production as  $x$  changes from 16 to 18 and as  $y$  changes from 81 to 80. Answer : \_\_\_\_\_

乙、計算、證明題：共 3 大題，每大題 12 分 (每小題 6 分)，共 36 分。須詳細寫出演算過程，否則不予計分。

1. (a) If  $f'(x) \leq 2$  for all  $x$ , what is the most the values of  $f$  can increase on  $[0, 6]$ ? Give reasons for your answer.

(b) Does the function  $f(x) = \begin{cases} \frac{1-\cos x}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$  have a derivative at  $x = 0$ ? Explain.

2. Evaluate (a)  $\int \cos(\ln x) dx$ . (b)  $\int_0^1 \int_x^1 y^2 e^{xy} dy dx$ .

3. Evaluate the following limits.

(a)  $\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x \tan^{-1} t dt$ . (b)  $\lim_{n \rightarrow \infty} \frac{1}{n} \left( \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{n\pi}{n} \right)$ .

參考用