國立中央大學97學年度碩士班考試入學試題卷

參考用

所別：資訊工程學系碩士班　科目：離散數學與線性代數　共 2 頁　第 1 頁
軟體工程研究所碩士班

※ 請務必按照題號次序寫在答案紙上，否則將嚴重失分。

1.(20%) Consider the following algorithm, which shuffles a list of integers. The algorithm assumes that the list size, n, is a power of 3.

```
1: shuffle( {a0, a1,..., an} )
2: if n == 1 return { a0 }
3: m = n/3
4: b = shuffle({a0,..., am-1 })        # the result is { b0,..., bm-1 }
5: c = shuffle({am,..., a2m-1 })       # the result is { c0,..., cm-1 }
6: d = shuffle({a2m,..., a3m-1 })      # the result is { d0,..., dm-1 }
7:
8: e = { c0,..., cm-1, d0,..., dm-1, b0,..., bm-1}
9:
10: return e
11 end shuffle
```

Answer the following questions according to the different assumptions:

(*a*)(10%) If line 8 use n copies to form the new list, what is the recurrence relation that counts the number of data copies for this algorithm? And estimate the complexity using big-θ notation.

(*b*)(10%) If instead the list is implemented by pointers and line 8 therefore only needs 3 pointers movements. What is the recurrence relation that counts the number of pointer movements for this algorithm? And estimate the complexity using big-θ notation.

2.(5%) What is the generating function for $\{a_k\}$, where a_k represents the number of ways to make change for k dollars using 1 dollor, 5 dollor, 10 dollor, and 20 dollors bills ?

3.(15%) Let $S = \{ 2, 3, 5, 7, 11, 13, 17, 19 \}$ be the set of prime numbers less than 20. If A is a subset of S, we can form the sum and product of the elements of A. For example, if $A = \{ 7, 11, 13 \}$, then the associated sum is $7 + 11 + 13 = 31$ and the associated product is $7 (11) (13) = 1001$.

(*a*)(8%) Use the Pigeon-Hole Principle to show that there are four nonempty subsets of S with the same sum.

(*b*)(7%) Are there two nonempty subsets of S with the same product ? Explain.

4.(10%) For integers a and b, define $a \sim b$ if $3a + 4b = 7n$ for some integer n.

(*a*)(5%) Prove that $\sim$ defines an equivalence relation.

(*b*)(5%) Find the equivalence class of 0.

（還有第二頁）

注意：背面有試題

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5.(10%) Find a subset of the following four vectors $v_1 = (1,0,1,1)$, $v_2 = (-3,3,7,1)$, $v_3 = (-1,3,9,3)$, and $v_4 = (-5,3,5,-1)$, that forms a basis for the space spanned by these vectors.

6.(10%) Find a basis for the nullspace of matrix A, where $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & 1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$.

7.(5%) Determine whether the vectors $v_1 = (1, -2, 3)$, $v_2 = (5, 6, -1)$, $v_3 = (3, 2, 1)$ form a linearly dependent set or a linearly independent set.

8.(18%) True and false (每小題答對給 3 分，答錯扣 3 分，不答 0 分；本題總分 ≥ 0)

(*a*) If square matrix A has one zero column, then A^TA is not diagonalizable.

(*b*) Every orthonormal set in R^n is linearly independent.

(*c*) If A is a square matrix with n orthonormal columns, then the rows of A must be an orthonormal basis for R^n.

(*d*) For an inconsistent linear system $Ax = b$, we can find its least-squares solution $(A^TA)^{-1}A^Tb$. However, if (A^TA) is not invertible, there is no least-squares solution.

(*e*) The quadratic form $2x_1^2 + 10x_1x_2 + 2x_2^2$ can be transformed into $7y_1^2 - 3y_2^2$ with no cross-product term.

(*f*) If A is a square matrix, then *det* A is the product of the singular values of A.

9.(7%) For a linear transform $x \mapsto Ax$ with $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$. Find a unit vector x at which Ax has maximum length and compute the length.

（題目到此為止）

注意：背面有試題