

科目：工程數學 A(5002)

參考用

校系所組：中大光電科學與工程學系、照明與顯示科技研究所

清大電機工程學系甲組、光電工程研究所

清大電子工程研究所、工程與系統科學系丁組

清大動力機械工程學系乙組

陽明醫學工程研究所醫學電子組、

陽明生醫光電工程研究所理工組 B

1. Consider the ODE $(3y^2 + x + 1)dx + 2y(x + 1)dy = 0$.

(1) (4%) Find an integrating factor for the ODE.

(2) (4%) Given $y(0) = 1$, solve the initial value problem.

2. Consider a mass-spring system governed by the ODE $y'' + 6y' + 18y = -90\sin(6t)$.

(1) (3%) How would you describe this system (choose one below)?

(A) Undamped; (B) Underdamped; (C) Critical damped; (D) Overdamped.

(2) (5%) Find the steady-state solution.

3. Consider the ODE $x^3y''' + 8x^2y'' + 9xy' - 9y = 0$ for $x > 0$.

(1) (5%) Find a basis of solutions $\{y_1(x), y_2(x), y_3(x)\}$ for the ODE.

(2) (4%) Given initial conditions $y(1) = 0$, $y'(1) = -2$, and $y''(1) = 2$, solve the initial value problem.

4. (5%) Bessel function of the first kind of order ν , $J_\nu(x)$, is one solution of the Bessel equation,

$x^2y'' + xy' + (x^2 - \nu^2)y = 0$. The general solution of the ODE, $x^2y'' + xy' + (4x^4 - \frac{1}{9})y = 0$, can be expressed as

$y(x) = C_1J_\nu(ax^2) + C_2J_{-\nu}(ax^2)$. Determine the values of a and ν .

5. (10 %) Use Laplace transform to solve $xy'' + (1-x)y' + ky = 0$.

(A) $y = \frac{e^t}{k!} \frac{d^k}{dt^k} [t^k e^{-t}]$ (B) $y = \frac{e^t}{k} \frac{d^k}{dt^k} [t^k e^t]$ (C) $y = \frac{e^t}{k} \frac{d^k}{dt^k} [t^k e^{-t}]$ (D) $y = \frac{e^t}{k!} \frac{d^k}{dt^k} [t^{-k} e^{-t}]$ (E) $y = \frac{e^{-t}}{k!} \frac{d^k}{dt^k} [t^k e^{-t}]$

(F) $y = \frac{e^{-t}}{k} \frac{d^k}{dt^k} [t^k e^{-t}]$ (G) $y = \frac{e^k}{t!} \frac{d^k}{dt^k} [t^k e^{-t}]$ (H) none of the above

6. (10 %) Find the Fourier transform of $f(x) = \sqrt{\frac{\pi}{2}}$ if $|x| < 2$ and $f(x) = 0$ otherwise.

(A) $f(w) = \frac{\sin w}{w}$ (B) $f(w) = \frac{\sin w}{2w}$ (C) $f(w) = \frac{\cos w}{w}$ (D) $f(w) = \frac{\cos w}{2w}$

(E) $f(w) = \sqrt{\frac{\pi}{2}} \frac{\sin w}{w}$ (F) $f(w) = \sqrt{\frac{2}{\pi}} \frac{\sin w}{w}$ (G) $f(w) = \frac{\cos 2w}{w}$ (H) none of the above

注意：背面有試題

參考用

7. Consider the problem

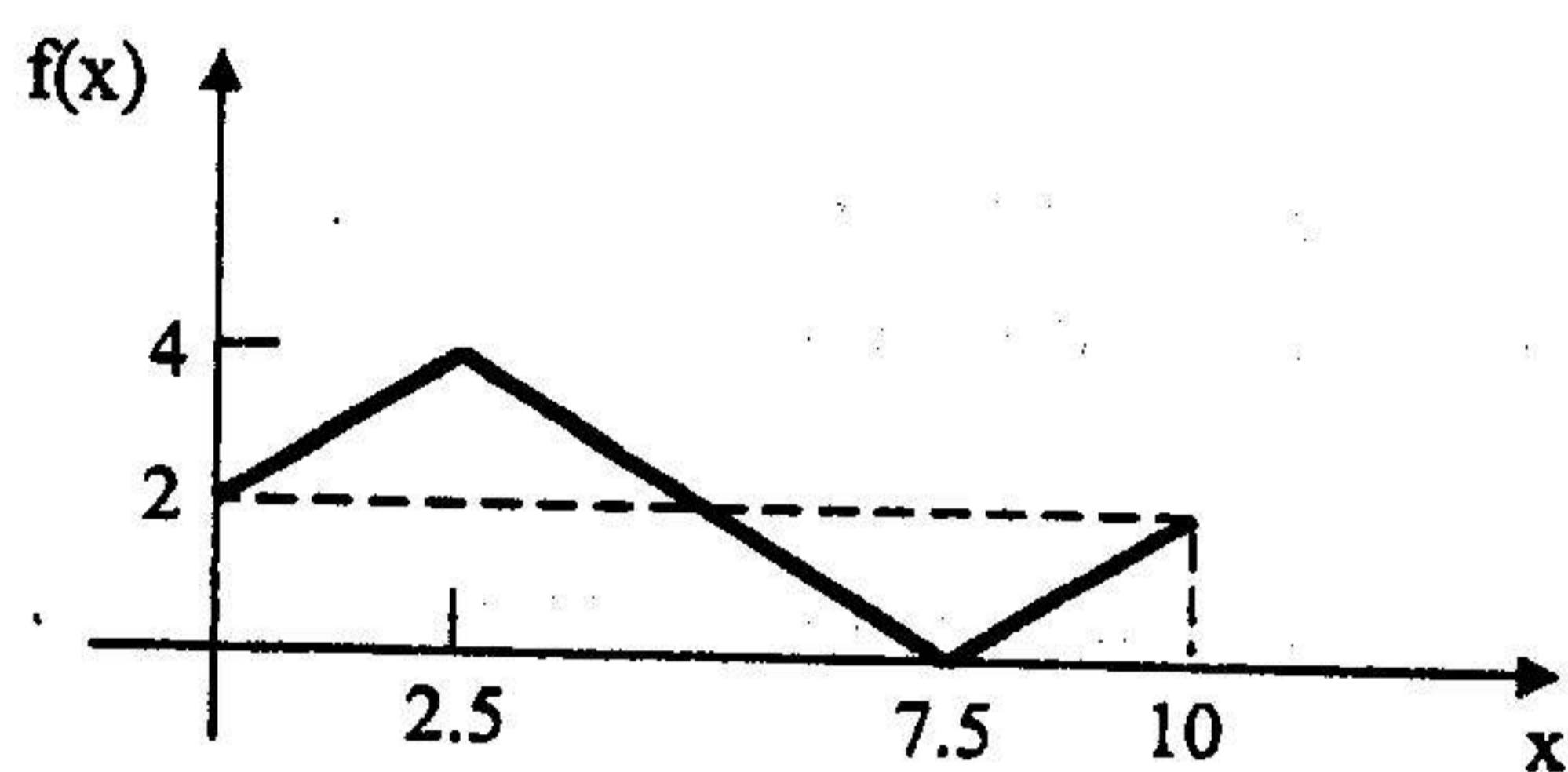
$$u_t - 4u_{xx} = 0 \quad 0 < x < 10$$

$$u(0, t) = u(10, t) = 2 \quad 0 < t$$

$$u(x, 0) = f(x) \quad 0 < x < 10$$

$$u_x(x, t = 0) = 0 \quad 0 < x < 10$$

$f(x)$ is shown in the following figure.



(1) (7%) What is $u(2, 1)$ (the value of u at position $x = 2$ when $t = 1$)?

- (A) 0.8 (B) 1.2 (C) 1.6 (D) 2 (E) 2.4 (F) 2.8 (G) 3.2
(H) none of the above.

(2) (6%) What is the lowest frequency (cycles per time) of the motion of u ?

- (A) 0.05 (B) 0.1 (C) 0.2 (D) 0.4 (E) 0.8 (F) 1.6 (G) 3.2
(H) none of the above.

8. (7%) The temperature distribution of a thin bar is described by a 1-D heat equation

$$u_t - 4u_{xx} = 0 \quad 0 < x < 10$$

The boundary and initial conditions are given as follows:

$$u(0, t) = u(10, t) = 0 \quad 0 < t$$

$$u(x, 0) = \sin \frac{\pi x}{10} \quad 0 < x < 10$$

The peak temperature is located at the position $x = 5$ at all time. At what time will the peak temperature reduce to $1/e$ of its initial value?

- (A) $\pi/10$ (B) $\pi^2/100$ (C) $10/\pi$ (D) $100/\pi^2$ (E) $\pi/5$
(F) $\pi^2/25$ (G) $5/\pi$ (H) none of the above.

9. (20%) Evaluate the principal value of the integral $\int_{-\infty}^{\infty} \frac{\cos 3x}{x^3 + x^2 + 3x - 5} dx$.

10. (10%) Find the eigenvalues and corresponding normalized eigenvectors (norm equals to 1) for the

matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix}$. What are those for the transpose matrix A^T ?