所別：統計研究所碩士班 一般生 學位在職生　科目：基礎數學　共 1 頁 第 1 頁

＊請在試卷答案卷（卡）內作答

1. (a) Prove that $\int_0^\pi \frac{dx}{\alpha - \cos x} = \frac{\pi}{\sqrt{\alpha^2 - 1}}$, $\alpha > 1$. (7%)

(b) Use (a) to prove $\int_0^\pi \ln \frac{b - \cos x}{a - \cos x} dx = \pi \ln \frac{b + \sqrt{b^2 - 1}}{a + \sqrt{a^2 - 1}}$, for $a > 1$ and $b > 1$. (8%)

2. Find the minimum and maximum values of $x^2 + y^2 + z^2$ subject to the constraint conditions $x^2/4 + y^2/5 + z^2/25 = 1$ and $z = x + y$. (10%)

3. Test for convergence: (a) $\sum_{n=1}^{\infty} \frac{4n^2 - n + 3}{n^3 + 2n}$ (b) $\sum_{n=1}^{\infty} \frac{n + \sqrt{n}}{2n^3 - 1}$ (c) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2 + 3}$. (5+5+5=15%)

4. Let $f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$. Prove that $\int_0^\pi f(x)dx = 2\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$. (10%)

5. (a) Verify, when A, D are symmetric matrices such that the inverses which occur in the expressions exist, that

$$\begin{pmatrix} A & B \\ B' & D \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} + FE^{-1}F' & -FE^{-1} \\ -E^{-1}F' & E^{-1} \end{pmatrix}$$ (10%)

where $E = D - B'A^{-1}B$, $F = A^{-1}B$.

(b) Find the inverse of

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 3 & 4 & 2 & 5 \end{pmatrix}$$ (10%)

6. Show that $\begin{vmatrix} A & C \\ B & D \end{vmatrix} = |A||D - BA^{-1}C|$, where A and D are square matrices and A is nonsingular. (10%)

7. Let A be an $n \times n$ matrix that is partitioned as follows (where $\det(A_{11}) \neq 0$):

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

If $\text{rank}(A) = \text{rank}(A_{11})$, show that $A_{22} = A_{21}A_{11}^{-1}A_{12}$. (10%)

8. If x_i is an $n \times 1$ vector for each $i = 1, 2, \ldots k$, and A is an symmetric matrix, show that

$$\text{tr}\left(A\sum_{i=1}^{k} x_i x_i'\right) = \sum_{i=1}^{k} x_i A x_i'$$ (10%)

參考用