

1. Solve the partial-differential equation

$$\alpha \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \quad 0 \leq x \leq L, \quad 0 \leq t,$$

with the conditions $T(0,t) = 0$, $T(L,t) = f(t)$, $T(x,0) = g(x)$. (25%)

2. Calculate $\int_0^{\infty} \frac{\cos(mx)}{(x^2+a^2)(x^2+b^2)} dx$ (25%)

3. Find the solution of $x^3 \frac{d^3 y}{dx^3} - 8x^2 \frac{d^2 y}{dx^2} + 55x \frac{dy}{dx} - 123y = x^3$; (25%)

4. Solve $X^2 - 5X + 3I = \begin{bmatrix} 1 & -4 \\ 2 & -5 \end{bmatrix}$. (25%)