1. The following partial differential equation with the boundary conditions

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y),\quad 0\le x,\ y\le 3,$$

$u(x,0)=g_1(x)$, $u(x,3)=g_2(x)$, $u(0,y)=h_1(y)$, $u(3,y)=h_2(y)$,

can be expressed as the following matrix form by the finite-difference method

$A\bar{U}=\bar{F}$,

where $x_0=y_0=0$, $x_1=y_1=1$, $x_2=y_2=2$, $x_3=y_3=3$, and

$\bar{U}=\{u(1,1),\ u(2,1),\ u(1,2),\ u(2,2)\}^T$.

Questions: What are the matrix A and the vector $\bar{F}$? (30%)

2. The Gaussian quadrature for the following integral is expressed as

$$\int_{-1}^{1} f(x)dx=\sum_{i=1}^{n} c_i f(x_i)$$

Questions: Using the Gaussian quadrature to calculate the following two integrals

(a) $\int_a^b f(x)dx$, (10%) and (b) $\iint f(x,y)dydx$, $0\le x^2+y^2\le a^2$. (30%)

3. The Runge-Kutta method of order 4 for the differential equation

$y'=f(t,y)$, $a\le t\le b$, $y(a)=\alpha$

is given by

$w_0=\alpha$, $k_1=hf(t_i,w_i)$, $k_2=hf(t_i+\frac{1}{2}h,w_i+\frac{1}{2}k_1)$, $k_3=hf(t_i+\frac{1}{2}h,w_i+\frac{1}{2}k_2)$,

$k_4=hf(t_{i+1},w_i+k_3)$, $w_{i+1}=w_i+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$.

Question: Extended the Runge-Kutta method for the following equations

$u_1'=f_1(t,\ u_1,\ ..u_n),\ ....,\ u_n'=f_n(t,\ u_1,\ ..u_n)$ with

$u_1(a)=\alpha_1,\ ....,\ u_n(a)=\alpha_n$. (30%)