

1. (25%) A random sample of 100 recorded deaths in a city showed an average life span of 71.8 years and a standard deviation of 8.9 years.
- (a) Estimate a 90% confidence interval for the mean life span of the city.
- (b) Test the hypothesis that the mean life span is greater than 70 years at the significance level of 0.05.
- (Hint: the critical value $z_{0.05}=1.645$ and $t_{0.05}=1.661$ with the 99 degrees of freedom)

2. (25%) Given N data points $(x_i, y_i), i = 1 \sim N$, we want to determine a circle that best describes the observed data. Formulate the least-squares circle fit to find the center and radius of the circle using a multiple linear regression model.

3. Consider a probability space consisting of the sample space $\Omega = \{(k, m) : k, m \in \mathbb{Z}^+\}$, i.e., all pairs of positive integers, where the set of events is the power set of Ω , and the probability measure is defined by assigning probabilities to points in the sample space such that:

$$P((k, m)) = p^2(1 - p)^{k+m-2}, \text{ for } 0 < p < 1$$

- (a) Find $P(\{(k, m) : k \geq m\})$. (10%)
- (b) Find the probability $P(\{(k, m) : k+m=r\})$ as a function of r for $r=2, 3, \dots$ (10%)
- (c) Find the probability $P(\{(k, m) : k \text{ is an odd number}\})$ (10%)

4. A number x is selected at random in the interval $[-1, 1]$. Let the event $A=\{x < 0\}$, $B=\{|x-0.5| < 1\}$, and $C=\{x > 0.75\}$.

- (a) Find the probabilities of B , $A \cap B$, and $A \cap C$ (10%)
- (b) Find the probabilities of $A \cup B$, $A \cup C$, and $A \cup B \cup C$ (10%)