---------- Some information ----------

Navier–Stokes equation:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

The thickness of laminar boundary layer can be approximated by: $\delta = 4.79\sqrt{\frac{x\nu}{U}}$;

Water's kinematic viscosity =1.12E-6 (m^2/s), density=1,000kg/m^3

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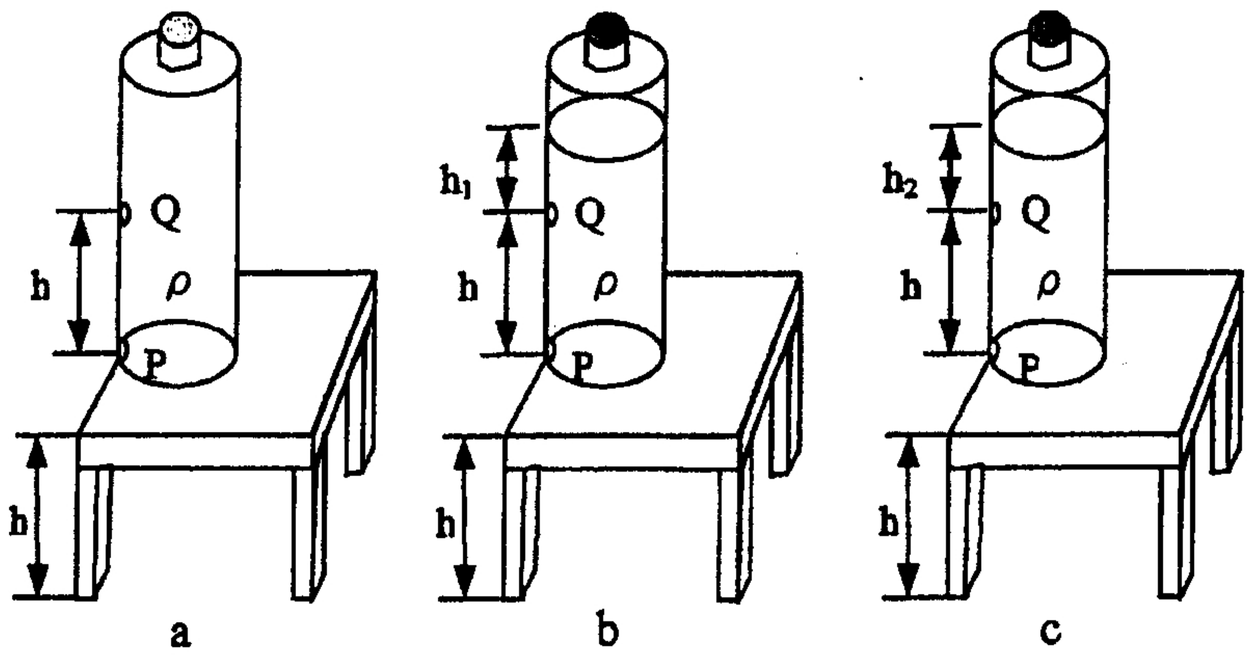
1. What is viscosity? How do you define a fluid's viscosity? (5%)

2. About potential flows (ideal fluid):
   A、 What are the assumptions of potential flows? (2%)
   B、 Please explain these assumptions. (5%)
   C、 If ideal fluid is not realistic, why do we still use it sometimes in engineering? (3%)

3. 如下圖，取一截面積爲 A_o 的 600c.c.圓柱形水瓶置於離地 h 的桌上，在瓶底挖一小孔 P，距底部高 h 處另挖一小孔 Q，兩孔的截面積均爲 A，而且 A_o 遠大於 A。我們先封住 P、Q 兩孔，將水瓶裝入密度爲 ρ 的純水後，鎖緊上方的瓶蓋，如下圖狀態 a。周圍的大氣壓力以 P_{atm} 表示，重力加速度爲 g。



(1) 若將 Q 孔及 P 孔同時打開，此時水會從 P 孔流出，空氣從 Q 孔流入並積在水瓶上方，如狀態 b 所示。此時水流出開孔 P 的水平初速爲何？(5%)

(2) 承上題，在瓶內水位降到高於 Q 孔 h_1 時，將 Q 孔堵住，最後 P 孔將不再有水流出時，如狀態 c 所示，水瓶內水位距離開孔 Q 的高度 h_2 與 h_1 比較，何者高？爲什麼？若不考慮表面張力，則水瓶上方內部的空氣壓力爲何？請說明理由。(10%)

4. Compare Euler equation and Navier–Stokes equation. (You do NOT need to write out the equations)
   A、 What is the difference? (5%)
   B、 What can these equations tell us about the fluid? (5%)
   C、 Each term in Navier–Stokes equation has a dimension of F/L^3 or $M/(T^2L^2)$ where M is mass, L is length, and T is time. But Navier–Stokes equation is also called "momentum equation". Why isn't its dimension the same as momentum? What physical meaning does each term in this equation represent? (10%)

5. 請舉一個 streamline、streakline、pathline 都不同的情形
   A、 上述三種線的中文爲何？他們是如何定義的？(9%)
   B、 畫圖標示此情形中的 streamline、streakline、pathline，並說明在此情形中他們爲何不同。(6%)
   C、 要如何改變此情形才會讓這三種線相同？(5%)

6. 我們考慮一個簡化的流體動力問題：有一長方體形狀之自主式水下載具於水中以 0.6m/s 等速前進，其長寬高分別爲 1.5m、15cm、20cm。
   A、 寫出 Reynolds number 的定義，並解釋其物理意義，再計算此水下載具的 Reynolds number。(3%+4%+3%)
   B、 計算在其艏後 30 公分處平板外殼的層流邊界層厚度（假設爲層流邊界層）。(4%)
   C、 上述邊界層假設爲層流邊界層是否合理？爲什麼？(4%)
   D、 若只考慮相對速度、流體黏滯性、物體之長寬高尺寸、流體密度、阻力等物理量，請將此問題作因次分析。(7%)
   E、 若其阻力係數爲 0.9，計算其推進器應提供之推力。 (5%)