

1. Vector Analysis:

(a) Let $F(x, y, z) = (y, 2xz, ze^x)$, compute $\text{div } F$ and $\text{curl } F$. (7%)(b) Let $f(x, y, z) = x^2 y \cos(yz)$, compute $\text{grad } f$. (6%)

(c) Evaluate the line integral (7%)

$$\int_C x dx - xy dy$$

If C is given by $x = t^2, y = -t; \quad 1 \leq t \leq 2$.

2. Find the general solution of the following differential equations:

(a) $y^2 dx + (2xy - x^4) dy = 0$ (6%)(b) $y'' - 2y' + y = e^x + x$ (7%)(c) $\mathbf{x}' = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -8 \\ 3 \end{bmatrix}$ (7%)

3. Given a $n \times n$ matrix A we wish to find a $n \times 1$ vector X so that $AX = Z$ for a given $n \times 1$ vector Z . List all conditions on A and Z in order that a solution may exist. (20%)

4. (20 points)

a). Let

$$f(z) = \frac{z^{1/2}}{(z+1)(z-2)^2}$$

please locate all the singularities and determine their nature.

b). Evaluate the integral

$$\frac{1}{2\pi i} \int_{|z|=3} \frac{dz}{(z^2+1)(z-2)^2(z-4)}$$

(背面仍有題目,請繼續作答)

5. (20%) With the help of the schematic diagram, solve the following two-dimensional boundary value problem within the square in term of the method of separation variable.

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

$$U = 1, 1 < x < 3, y = 1$$

$$U = 0, 1 < x < 3, y = 3$$

$$U = 0, x = 1, 1 < y < 3$$

$$U = 0, x = 3, 1 < y < 3$$

