

1. Evaluate $I = \oint_C \cot(z) dz$, where C is the unit circle $|z|=1$ traversed in a clockwise sense. (15%)

2. Let $f(x) = x^2/2$ for $-0 \leq x \leq \pi$. Find the Fourier series of $f(x)$ and evaluate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (15%)

3. Find the general solution of the equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dt} + 2y = \delta(x-3)$. (20%)

4. Solve $\dot{X} = AX$, where $X^T = [x_1 \ x_2]$, $\dot{X} = dx/dt$,

$A = \begin{bmatrix} 1 & 3 \\ -3 & 7 \end{bmatrix}$, the superscript T denotes transpose of a vector or matrix. (15%)

5. Let $F(x, y, z) = (-y+z, x+yz, xyz)$. By applying Stokes' Theorem, compute the integral of $\text{Curl } F$ over the hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$, with outwards normals. (15%)

6. Find the general solution of $y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1$, $x > 0$,

where $y' \equiv dy/dx$, $y'' \equiv d^2y/dx^2$.

(20%)