1. Let X and Y be random continuous variables with finite moments.

   Please show that var(Y)= $E_X\left[\text{var}(Y|X)\right]+\text{var}_X\left[E(Y|X)\right]$ (10%)

2. Let $\bar{X}$ and S^2 be the sample mean and sample variance of a random sample $X_i, i=1,2,3\ldots n$ from a population with finite mean μ and variance σ^2. Prove the following properties:

   (a) $\bar{X}$ and $(X_i-\bar{X})$ are uncorrelated for each i. (10%)

   (b) $\bar{X}$ is the BLUE of μ. (10%)

3. Consider the linear model,

$$Y_i=\beta X_i+\varepsilon_i \quad \text{for } i=1,2\ldots,N$$

where Y_i are independent random variables which you can observe from the experiment, ε_i are independent random variables with zero mean and variance is $\text{var}(\varepsilon_i)=\sigma^2$ for all i. In order to estimate β, consider the following two alternative estimators.

First estimator : $b_{(I)}=\dfrac{\sum_{i=1}^{N}y_i x_i}{\sum_{i=1}^{N}x_i^2}$

Where $y_i=Y_i-\bar{Y}$, $x_i=X_i-\bar{X}$

Second estimator :

You randomly choose two points from the sample of size N, and draw a line between these two points. Algebraically, if these two points are (Y_1,X_1) and (Y_2,X_2), then this second estimator is : $b_{(II)}=\dfrac{Y_2-Y_1}{X_2-X_1}$

(a). Show that $b_{(I)}$ and $b_{(II)}$ are unbiased estimators. (15%)

(b). Find the variances of $b_{(I)}$ and $b_{(II)}$. (15%)

（背面仍有題目,請繼續作答）

4. The Geometric distribution is known as the probability of waiting for the first success in independent repeated trials of a Bernoulli process. This could occur on the 1^{st}, 2^{nd}, 3^{rd} …trials.

$$g(X;\theta)=\theta(1-\theta)^{X-1} \qquad \text{for } X=1,2,3,\ldots$$

Given a random sample from this geometric distribution of size n, find the MLE (maximum likelihood estimator) of θ and the method of moments estimator of θ. (20%)

5. A stock market analyst wants to estimate the relationship between EPS (earning per share) and the number of meetings hold by the managers. He has 30 firms as the samples and uses the following regression model,

$$y_i=\beta_0+\beta_1 x_i+u_i \qquad i=1,2,3..,30$$

where y_i denotes EPS of firm i and x_i denotes the number of meetings. The analyst calculates the following information,

$$\sum_{i=1}^{n}(x_i-\bar{x})^2=100 \qquad \sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})=120$$

$$\sum_{i=1}^{n}\hat{u}_i^{\,2}=56 \qquad n=30$$

(a) Compute the OLS estimator of $\hat{\beta}_1$.(3%)

(b) Compute the standard error of regression. (5%)

(c) Computer R^2.(7%)

(d) Computer the variance of OLS estimator $\hat{\beta}_1$. (5%)