(1) Scratch a simple diagram to explain the geometrical meanings of the following quantities: (a) $\vec{A}\cdot(\vec{B}\times\vec{C})$, (b) $\nabla\varphi$, (c) $\nabla\cdot\vec{A}$, (d) $\nabla\times\vec{A}$. (15%)

(2) Evaluate $\iint\nabla\times(y\hat{i}+3\hat{j}+5\hat{k})\cdot\hat{n}d\sigma$, where σ is the surface in the first octant made up of part of the plane x+2y+3z=6, and triangles in the (x, z) and (y, z) planes. (10%)

(3) (a) Solve $dN/dt+aN=e^{-\beta t}$, where a, β are constants. (5%) (b) Find the general solution of the differential equation $d^2x/dt^2+5\,dx/dt+4x=2\cos(2t)$, and give some discussions on the physical meaning of the complementary function and the particular solution. (10%)

(4) Find the Fourier series representation of function

$$f(t)=\begin{cases}0, & -\pi\le\omega t<0\\ \sin\omega t, & 0\le\omega t<\pi\end{cases}.(10\%)$$

(5) (a) Prove the Cauchy's integral formula $\oint_C\frac{f(z)dz}{(z-a)}=2\pi i\,f(a)$ by using Cauchy's theorem $\oint g(z)dz=0$, where $f(z)$ and $g(z)$ are analytical function inside the contour C (8%) (b)Evaluate the definite integral $\int_0^\infty\frac{dx}{(4x^2+1)^3}$. (7%)

(6) Find the eigenvalues and eigenvectors of the matrix $M=\begin{pmatrix}1 & 0 & 3\\ 0 & -2 & 0\\ 3 & 0 & 1\end{pmatrix}$. (15%)

(7) In the initial steady state of an infinite slab of the thickness d, the face x=0 is at 0 ℃ and the face x=d is at T_0. From t=0 on, the x=0 face is held at T_0 and the x=d face at 0℃. Find the temperature distribution at time t, T(x,t). (Note: T(x,t) obeys the diffusing equation $\nabla^2T(x,t)=\frac{1}{\alpha^2}\frac{\partial T(x,t)}{\partial t}$.)(20%)