1. A real square matrix is shown as $A=\left[a_{jk}\right]$, which transpose matrix and inverse matrix are $A^T$ and $A^{-1}$, respectively.

(a) Please answer what relations must be satisfied among $A$, $A^T$ and $A^{-1}$ when matrix A is symmetric, skew-symmetric, or orthogonal, respectively. (5%)

(b) If matrix A is shown as $A=\begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}$, please find $e^{At}$? (10%)

2.

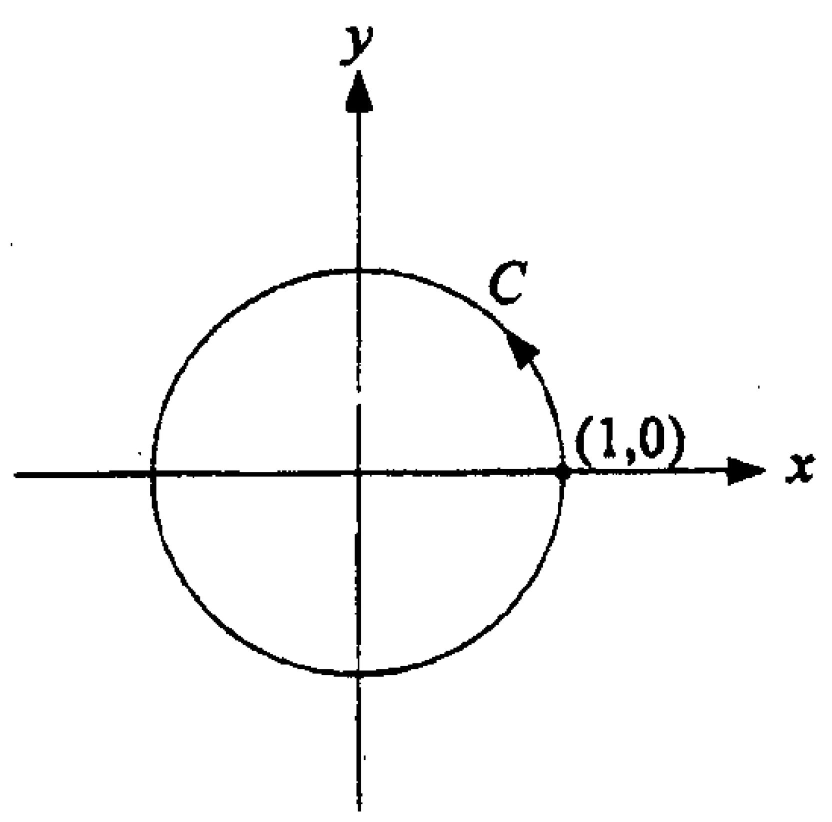
(a) Find the Fourier series representation of

$$f(x)=\begin{cases} 0, & -\pi < x \le 0 \\ x, & 0 \le x < \pi \end{cases} \qquad (10\%)$$

(b) From the Fourier expansion show that

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots. \qquad (10\%)$$

3. Please apply Green's theorem to evaluate $\oint_C (3x\,dy-5y\,dx)$, the contour $C$ is a circle and shown below. (15%)




4. The differential equation $m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$ can be used to describe a damped simple harmonic motion. Its solution can be written as the form of $x(t)=x_m e^{-\alpha t}\cos(wt+\phi)$, where $x_m$ is the amplitude of the damped oscillator. Please solve this differential equation and find the $\alpha$ and $w$ in terms of $m$, $b$, $k$ (20%).

（背面仍有題目,請繼續作答）

5. The binomial distribution is $P(m) = \frac{n!}{m!(n-m)!} p^m (1-p)^{n-m}$. In the limit $n \to \infty$, $p \to 0$, and $np = a$, find the new distribution $P(m)$ (Hint: use $\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n = e^{-a}$) (10%)

6. Using theorem of residues, calculate $\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{i\omega t}}{\omega_0^2 - \omega^2 + i\alpha\omega}$ $(\alpha > 0)$ for (a) t<0 (b) t>0 (20%)