1. Evaluate $\int_0^{\frac{\pi}{4}} x \sec x \tan x \, dx$. (10%)

2. Evaluate $\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)}$. (10%)

3. (a) Show that $\int_b^a x^y \, dy = \frac{x^b - x^a}{\ln x}$, where $a > 0$, $b > 0$. (5%)

   (b) Evaluate $\int_0^1 \frac{x^b - x^a}{\ln x} \, dx$. (5%)

4. Find $\lim_{n \to \infty} \frac{1}{n^{\frac{3}{2}}} \left(\sqrt{n} + \sqrt{n+1} + \cdots + \sqrt{2n-1} \right)$. (10%)

5. (a) Find the Maclaurin series for e^x. (5%)

   (b) Find the $(2n+1)$th-order Taylor polynomial to approximate the function

$$E(x) = \int_0^x e^{-t^2} \, dt.$$ (5%)

6. From Kepler's laws $r = \frac{1}{C - D\cos\theta}$ and $r^2 \frac{d\theta}{dt} = h$, show that the inverse square law,

$$\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -C\frac{h^2}{r^2}.$$ (10%)

7. Find an equation of the tangent line to the graph of the equation $\ln(xy) - 2x - y = 3$ at the point $(-\frac{1}{2}, -2)$. (10%)

8. The equation $x^2 + 2y^2 + 3z^2 + xy - z - 9 = 0$ defines z implicitly as a function of x and y. Find the values of $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial z}{\partial y^2}$ at the point $(1, -2, 1)$. (10%)

9. The temperature at each point of the wire $x^2 + y^2 = 1$ is $T = x^2 + 2y^2 - x$. Find the hottest and coldest point of the wire. (10%)

10. For finding the volume of the region above the xy plane bounded by the surface $x^2 + y^2 = 1$ and $z = x$,

    (a) list the double integration in rectangular coordingates, (3%)

    (b) list the double integration in polar coordingates, (3%)

    (c) evaluate one of the double integration in (a) and (b). (4%)