本試題是否可以使用計算機：□可使用 ，☑不可使用 （請命題老師勾選）　　考試日期：0301，節次：2

1. Find $\lim_{n\to\infty} \sum_{k=n+1}^{2n} (\ln k - \ln n)$. (10%)

2. The function

$$f(x) = \begin{cases} e^{\frac{-1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Find the derivative of $f(x)$. (10%)

3. Evaluate $\int_0^{\frac{\pi}{6}} \frac{\sin x \cos x}{1 - \sin x}\, dx$. (10%)

4. Evaluate $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, dx$. (10%)

5. (a) Find the Maclaurin series for $\sin x$. (5%)

(b) Explain why $\left|\frac{\sin x}{x} - 1\right| \leq \frac{1}{6}|x^2|, \quad \forall\, x \neq 0$. (5%)

6. Find an equation of the tangent lin to the graph of the equation $\tan^{-1}\frac{y}{x} = \ln\sqrt{x^2+y^2}$ at the point $(1,0)$. (10%)

7. A wire is the form of the unit circle $x^2 + y^2 = 1$ is heated in such a way that its temperature at (x,y) is $T = xy$. Find the hottest and coldest points of the wire. (10%)

8. Let R be the region $|x| + |y| \leq 1$.

(a) Show that $\iint_R f(x+y)\, dA = \int_{-1}^{1} f(u)\, du$, where $u = x + y$. (5%)

(b) Evaluate $\iint_R \log(1 + x + y)\, dA$. (5%)

9. Sketch the graph of the equation $y = x^4 - 6x^2 + 8x + 10$ by discussing its significant features. (i.e. relative extreme, inflection points ....) (10%)

10. Find the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 2x$. (10%)