15%(1): In Fig.1, a solid uniform sphere of mass m, radius R is moving down on a wedge of mass M, Suppose the motion of the sphere is rolls without slipping, and there is no friction between the wedge and the floor, therefore the wedge will also moves while the sphere moving down. Find the accelerations of the center of the sphere and the wedge.

15%(2): Consider a rectangular plate of mass M, and sides a and b, choose the two sides as the x and y axis(see Fig.2).

(a) Find the inertia tensor of the plate about the chosen coordinate.(3%)

(b) Find the principal moments of inertia and the directions of the principal axes for the plate.(4%)

(c) If the plate rotate about a diagonal $\overline{OA}$ with a constant angular velocity ω, what is the kinetic energy of the plate.(4%)

(d) In part (C), find the torque need to rotate the plate.(4%)

20%(3): In Fig.3, $\overline{AB}$ is a straight frictionless wire fixed at point A on a vertical axis $\overline{AO}$ such that $\overline{AB}$ rotates about $\overline{AO}$ with constant angular velocity ω, and θ be the angle between $\overline{AB}$ and $\overline{AO}$, which is a constant. A bead of mass m is constrained to move on the wire, let r be the distance of the bead to point A, and choose r as the generalized coordinate.

(a) Write down the transformation between the generalized coordinate and the rectangular coordinate. (2%)

(b) Write down the kinetic energy and the potential energy in terms of generalized coordinate r and generalized velocity $\dot{r}$, and then find the Lagrangian of the bead, what is the generalized momentum P_r.(4%)

(c) Write down the Hamiltonian of the bead, is the Hamiltonian equal to the total energy of the bead? If not, why? and what are the canonical equations of motion?(5%)

(d) By using of Lagrange's equations of motion, find the equation of motion of r.(5%)

(e) Solve the equations in part (d), show that

$$r(t) = Ae^{(\omega \sin\theta)t} + Be^{-(\omega \sin\theta)t} - \frac{g\cos\theta}{\omega^2 \sin^2\theta}$$

where A, B are constants determined by initial conditions.(4%)

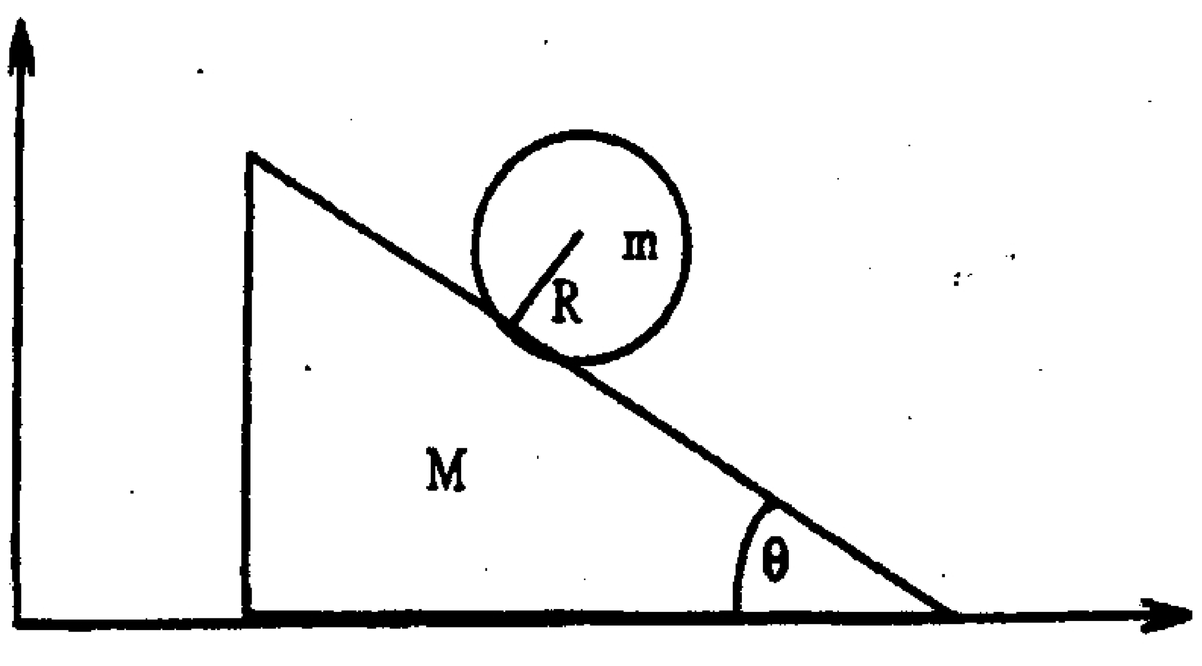

Fig-1

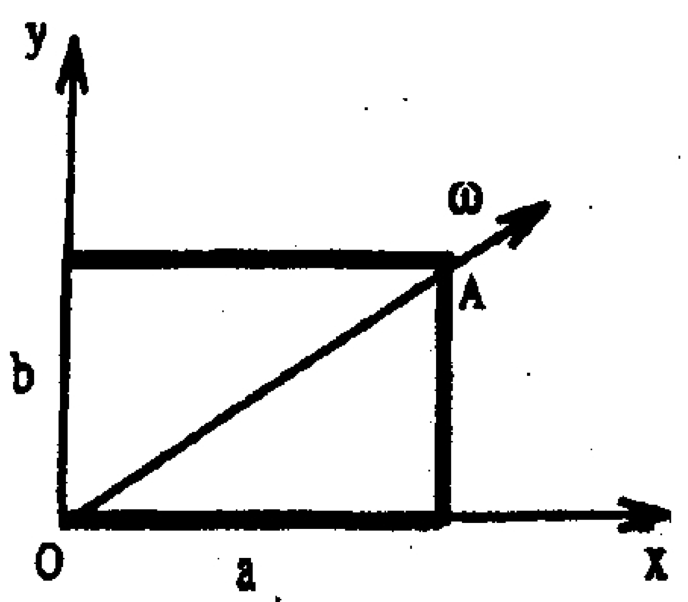

Fig-2

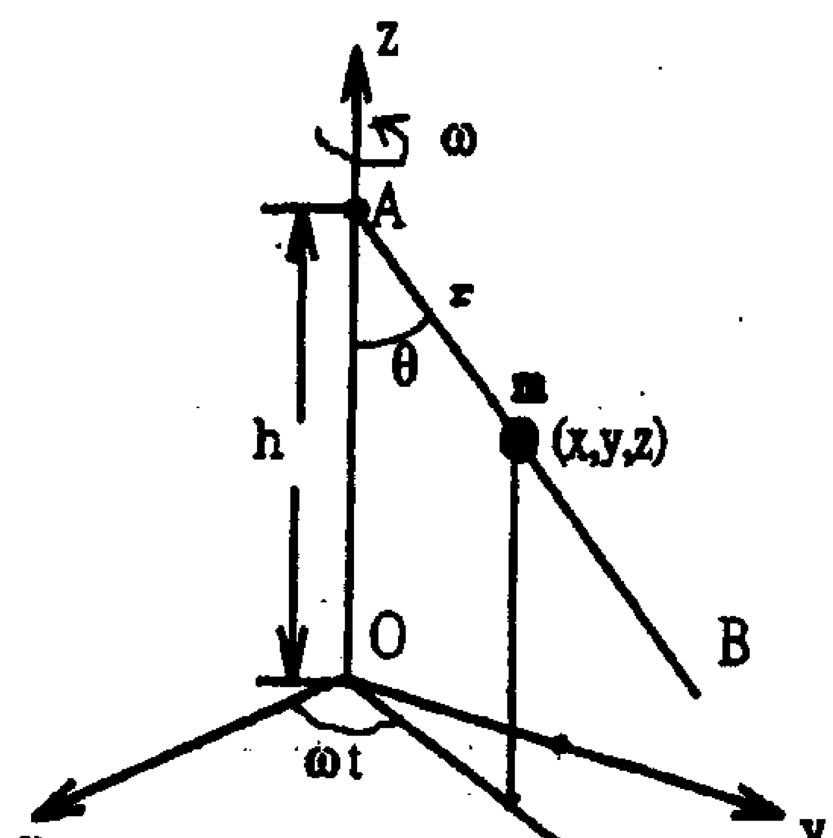

Fig-3

（背面仍有題目,請繼續作答）

編號： 49 國立成功大學九十七學年度碩士班招生考試試題 共 2 頁，第2頁

系所：物理學系 科目：古典力學

本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選） 考試日期：0301，節次：1

15%(4): A particle of mass m when at infinity has kinetic energy T_o, and is incident into a rigid frictionless sphere of radius a with impact parameter b(see Fig.4), the mass of the sphere is also m, and initially at rest. The potential energy may be expressed as:

$$U(r) = \begin{cases} \infty & \text{for } r \leq a \\ 0 & \text{for } r > a \end{cases}$$

(a) Show that in the center of mass system, the scattering angle of the particle is $\theta = 2\cos^{-1}\dfrac{b}{a}$, and what is the scattering angle in the laboratory system?(7%)

(b) Calculate the differential cross section σ and the total cross section σ_t in both center of mass system and laboratory system.(8%)

20%(5): Consider a linear triatomic molecules CO_2 as shown in Fig.5, m and M are the mass of carbon atom and oxygen atom respectively, and k is the force costant between two atoms, if the motion of the molecule is constrained in x direction, find the normal frequencies of the motion, and what are the corresponding normal modes? Can you explain the normal modes by using of conservation of linear momentum?

15%(6): (a) Show the wave equation for the small transverse vibration of a string along x axis is:

$$T\frac{\partial^2 y}{\partial x^2} = \rho\frac{\partial^2 y}{\partial t^2}$$

where y is the disturbance of the string at x, and T is the tension of the string, which suppose to be a constant, and ρ is the mass density of the string.(5%)

(b) A sinusoidal wave $y = A\cos(\omega t - kx)$ propagate in a uniform string, show that

$$k = \omega\sqrt{\frac{\rho}{T}}$$

Is the medium dispersive? why?(5%)

(c) An incident wave traveling along $+x$ direction on a string with mass density ρ_1, represented by $y = A\cos(\omega t - k_1 x)$. At $x = 0$, the wave transmitted into another string with different mass density ρ_2, part of wave reflected and the part of the wave transmitted, the amplitude of reflected wave and transmitted wave are B and C respectively. By using the boundary conditions at $x = 0$, show that

$$\frac{B}{A} = \frac{\sqrt{\rho_1} - \sqrt{\rho_2}}{\sqrt{\rho_1} + \sqrt{\rho_2}}, \quad \text{and} \quad \frac{C}{A} = \frac{2\sqrt{\rho_1}}{\sqrt{\rho_1} + \sqrt{\rho_2}} \quad (5\%)$$

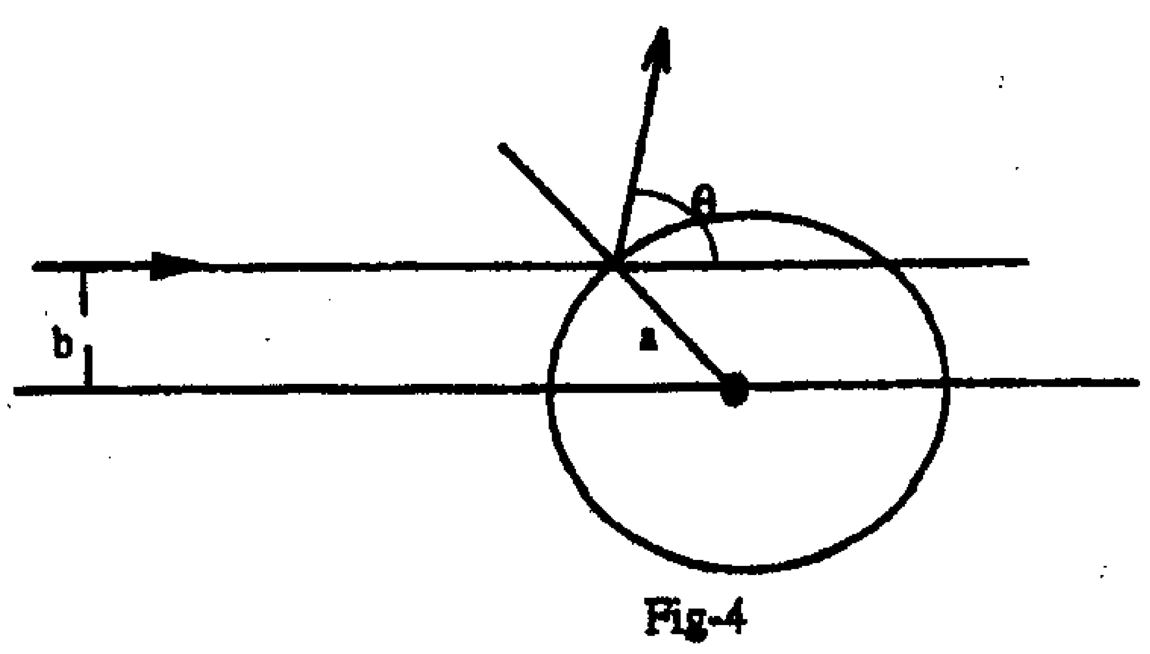

Fig-4

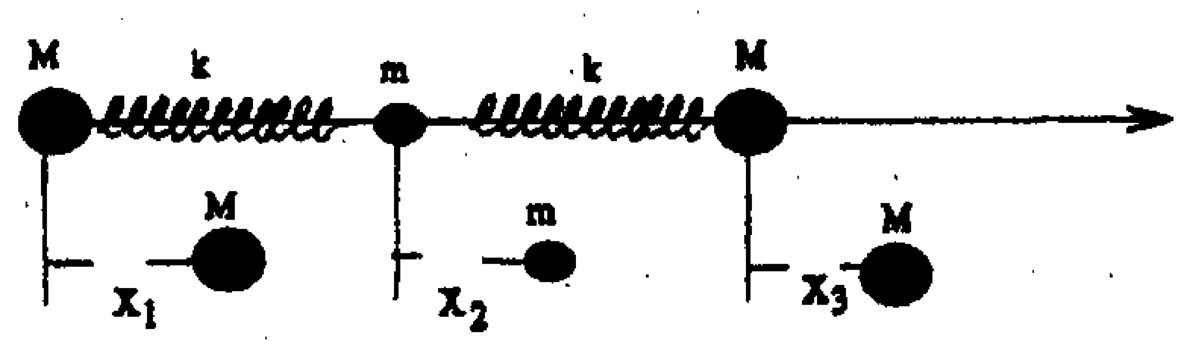

Fig-5