1. (15%) Solve $x = yp + p^2$ where $p \equiv \dfrac{dy}{dx}$.

2. (15%) Solve $(D_x^3 + D_x^2 D_y - D_x D_y^2 - D_y^3)z = (D_x + D_y)^2(D_x - D_y)z = e^x \cos 2y$, where $D_x \equiv \partial/\partial x$ and $D_y \equiv \partial/\partial y$.

3. (20%) Classify according to type (elliptic, hyperbolic or parabolic?) and determine the characteristics of
   (a) $2u_{xx} - 4u_{xy} - 6u_{yy} + u_x = 0$
   (b) $u_{xx} - x^2 y u_{yy} = 0 \quad (y > 0)$

4. (20%) Determine the following statements are true (T) or false (F). (Need not to state reasons.)
   (a) For two square matrices A and B, if $AB = I$, the identity matrix, then $BA = I$.
   (b) For a negative-definite matrix M, its determinant $\det(M) < 0$.
   (c) The set of all 5×4 matrices is a vector space.
   (d) For a system of linear equations, $A\mathrm{x} = \mathrm{b}$, if it has no exact solution, then it has a unique least-squares solution.

5. (20%) Consider a matrix A defined by its eigen-decomposition (or diagonalization) as follows,

$$A = EDE^{-1} = \begin{bmatrix} 5 & 4 & 0 & 0 \\ 6 & 5 & 0 & 0 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 5 & 4 & 0 & 0 \\ 6 & 5 & 0 & 0 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 4 & 3 \end{bmatrix}^{-1},$$

where the columns of matrix E are the eigenvectors of A. Find the eigenvalues and the corresponding eigenvectors of A^T.

6. (10%) Let M be an $n \times n$ real matrix with orthogonal columns, and the Euclidean norm of each column is 1. Find the determinant of M.