

1. Find the curve $y(x)$ that passes through $(1, 0.5)$ and is such that at each point (x, y) the intercept of the tangent on the y -axis is equal to $2xy^2$. (15%)

2. Solve the following initial value problems.

(a) $x^2 y'' - 4xy' + 4y = 0$, $y(1) = 4$, $y'(1) = 13$. (10%)

(b) $(x^2 D^2 - 5xD + 8)y = 0$, $y(1) = 5$, $y'(1) = 18$. (10%)

3. Compute

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx \quad (15\%)$$

4. What is the order of the pole at $z = 0$ of the following function? Why?

$$f(z) = \frac{1}{(2 \cos z - 2 + z^2)^2} \quad (15\%)$$

5. Find the Singular-Value Decomposition (SVD) of the matrix

$$A = \begin{bmatrix} 5 & 0 & 1 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 4 & 0 & -4 \end{bmatrix}. \quad (15\%)$$

6. Let $f(t) = e^{-|t|}$, and $g(t) = \begin{cases} 1, & -1 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$ (20%)

(a) Compute $y(t) = f(t) * g(t)$, where $*$ denotes convolution.

(b) Find the Fourier transform of $y(t)$.