

1. (20 pts). Let  $\vec{a} = (1,1,0)$ ,  $\vec{b} = (1,0,1)$  be  $3 \times 1$  vectors in the Euclidean space. In addition, let  $\vec{c}$  be a  $3 \times 1$  vector in the Euclidean space. Assume that  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  has the following relationship:  $\vec{c} \cdot (\vec{a} \times \vec{b}) = 0$ . Give the solution set of  $\vec{c}$ . (Note: The solution set may contain one or more than one solutions of  $\vec{c}$ .)
  
2. (20 pts) The Euler formula:  $e^{i\theta} = \cos \theta + i \sin \theta$ .
  - (a) (10 pts) Apply the Euler formula to derive the following equality:
 
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
  - (b) (10 pts) Apply the Euler formula to derive  $d \cos \theta / d\theta$  and  $d \sin \theta / d\theta$ .  
**Note:** no credit will be given unless the Euler formula is applied in the derivation.
  
3. (20 pts) Find the following areas.
  - (b) (10 pts) Let  $A$  be the area of the region bounded above by  $y = x + 2$  and below by  $y = x^2$ . Find  $A$ .
  - (a) (10 pts) Let  $B = \int_{-1}^1 \frac{1}{x^2} dx$ . Give  $B$ .
  
4. (20 pts) Solve the following problems.
  - (a) (10 pts) Let  $y = 1/\ln x$ . Give  $dy/dx$ .
  - (b) (10 pts) Let  $y = \int_3^{x^2+x} \frac{1}{t^3+1} dt$ . Derive  $dy/dx$ .
  
5. (20 pts) Evaluate the following limits:
  - (a) (5 pts)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{c})^n = ?$ , where  $c > 0$ .
  - (b) (5 pts)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^c = ?$ , where  $c > 0$ .
  - (c) (10 pts) Is it true that  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = 1$ ? Explain your answer.