1. Name five important persons and briefly describe their contributions in the development of mathematical programming. (15%)

2. For the following linear program with bounded variables, formulate its dual.
   max. $c^t x$
   s.t. $Ax \le b$
   $l \le x \le u$
   where A is a constant matrix, c, b, l, and u are constant column vectors, and x is a variable column vector. (15%)

3. For the following mathematical program:
   max. $x_1 + 2x_2$
   s.t. $x_1 + x_2 \le 4$
   $3x_1 + 2x_2 - |2x_1 - x_2| \le 6$
   $x_1, x_2 \ge 0$
   (a) Transform it into a conventional linear program which can be solved by LP computer packages. (10%)
   (b) Use any method to solve. (10%)

（背面仍有題目,請繼續作答）

4. (30%) Consider a workshop consisting of two machines. The potential jobs arrive in accordance with a Poisson process at rate 2, and that the service times for the two machines are independent and have respective exponential rates of 2 and 3. Suppose that an entering job first will processed on machine 1. When its work is completed on machine 1, it will go either machine 2 if that machine is idle or else wait in machine 1 until machine 2 becomes available. Suppose that a potential job will enter this workshop as long as machine 1 is idle.

   (a) What proportion of potential jobs enters the workshop? (10%)

   (b) What is the mean number of jobs in the workshop? (10%)

   (c) What is the average amount of time that an entering job spends in the workshop? (10%)

5. (10%) Trial are performed in sequence. If the last two trials were successes, then the next trial is a success with probability 0.8; If the last two trials were fail, then the next trial is a success with probability 0.3; Otherwise the next trial is a success with probability 0.5. In the long run, what proportion of trials are success?

6. (10%) Let p_{ij}'s be the one step transition probabilities of a Markov chain with $M+1$ states. If this Markov chain is irreducible, aperiodic, and

$$\sum_{i=0}^{M} p_{ij} = 1, \text{ for all } j.$$

   Show that the limiting probabilities

$$\pi_j = \frac{1}{M+1}, \text{ for } j = 0, 1, \ldots, M.$$