國立交通大學 97 學年度碩士班考試入學試題

科目：作業研究(5094)　　考試日期：97 年 3 月 9 日 第 3 節

所班別：工業工程與管理學系　　組別：工工管系乙組　　

【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (18%) Consider the transportation problem having the following parameter table:

		Destination				Supply
		W	X	Y	Z	
Source	A	1	3	2	1	15
	B	3	1	2	1	20
	C	1	1	3	2	20
Demand		18	12	15	20	

(a) (4%) Use Russell's approximation method to obtain an initial basic feasible solution.
(b) (10%) Apply the **transportation simplex** method, starting with the initial solution obtained in (a), to obtain an optimal solution.
(c) (4%) Formulate the transportation problem as a linear programming problem.

2. (16%) Minimize $Z = 5x_1 + 8x_2 + 7x_3 + 4x_4 + 6x_5$

Subject to $2x_1 + 3x_2 + 3x_3 + 2x_4 + 2x_5 \le -20$

$3x_1 + 5x_2 + 4x_3 + 2x_4 + 4x_5 \le 30$

$x_j \ge 0, \quad j=1,\ldots,5$

(a) (12 %) Apply **revised simplex** method to solve the problem. Show all your steps in detail.
(b) (4 %) Write the dual of the prime model.

3. (20%) The management of a company is considering three possible new products for next year's product line. A decision now needs to be made regarding **which products to market and at what production levels**.

Initiating the production of two of these products would require a substantial start-up cost, as shown in the first row of the table below. Once production is under way, the marginal net revenue from each unit produced is shown in the second row. The third row gives the percentage of the available production capacity that would be used for each unit produced.

	Product		
	1	2	3
Start-up cost, c_n	3	2	0
Marginal net revenue, r_n	2	3	1
Capacity used per unit, %	20	40	20

Only 3 units of product 1 could be sold, whereas all units that could be produced of the other two products could be sold. The objective is **to determine the number of units of each product to produce in order to maximize the total profit** (total net revenue minus start-up costs).

(a) (10%) Assuming that production quantities must be integers, use dynamic programming to solve this problem. (Hint: At stage n, let the state s_n be measured as the number of 20% blocks of production capacity still available for use on the remaining products, so the possible values of s_n are 0, 1, 2, 3, 4, 5.)

(b) (10%) Now consider the case where the divisibility assumption holds so that the variables representing production quantities are treated as continuous variables. Assuming that proportionality holds for both net revenues and capacities used, use dynamic programming to solve this problem. (Note that the units of s_n now are different than in part (a), so the possible values of s_n now are $0 \le s_n \le 100$ rather than 0, 1, 2, 3, 4, 5.)

4. (13%) Company ABC invested \$15,000 in the design and development of a new product. The unit production cost of the new product is \$2. The market manager gives the conclusions of market research as follows: if company ABC spends \$ a on advertising and sell the product at price p (per unit), it will sell $2,000 + 4\sqrt{a} - 20p$ units. Using this figure, express the profit that company ABC will make as a function of a and p. What price and level of advertising will maximize its profits?

5. (18%) A taxi company has divided the city into three areas – Downtown, the North End and the South End.

Passengers picked up on the North End: 50% stay in the region, 20% go to the Downtown, and 30% go to the South End;

Passengers picked up in the Downtown: 10% go to the North End, 40% stay in the Downtown, and 50% go to the South End;

Passengers picked up on the South End: 30% go to the North End, 30% go to the Downtown, and 40% stay on the South End.

(a) (6%) If a taxi driver starts his work in the Downtown, what is the probability that he will be in the Downtown after getting his 3rd fare?

(b) (6%) The taxi driver noticed that fares picked up in the Downtown yield on average \$100, in the South End \$120, and in the North End \$150. He also noticed that on average he has 15 pick-ups per day. What is his expected total earning in a month (30 days)?

(c) (6%) Suppose that there are some bad guys in the area of the North End and they trap any taxi coming to the North End. The taxi driver, who is unaware of this, starts his work in the Downtown. How many times of fares he is expected to pick before he gets trapped?

6. (15%) Consider an M/M/1 system with the arrival rate λ, and the service rate μ where customers are impatient. Specifically, upon arrival, customers estimate their queueing time w and then join the queue with probability $e^{-\alpha w}$. The estimate is $w = k/\mu$ when the new arrival finds k customers in the system. Assume $0 \le \alpha$.

(a) (5%) Find the actual arrival rate λ_k and service rate μ_k when the new arrival finds k customers in the system?

(b) (10%) Suppose $\alpha \to \infty$, find the average number of customers in the system. (Hint: you may need to find the probability that the new arrival finds k customers in the system.)