國立交通大學 97 學年度碩士班考試入學試題

目：工程數學(3123)　　考試日期:97 年 3 月 8 日 第 1 節

所班別：土木工程學系　　組別：土木系戊組　　第 1 頁,共 1 頁

【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. Given $P \in R^{n\times n}$，$A \in R^{n\times u}$ and P is a positive definite matrix, find the rank of the matrix $(\mathbf{P - PA(A^TPA)^{-1}A^TP})$ (15%)

2. The following recurrence relation holds for the Hermite polynomial

$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$

where n is degree. The first two polynomials are $H_0 = 1, H_1 = 2x$. Derive $H_6(x)$ (20%)

3. We call the quantity

$$D_n(x) = \frac{\sin\left(n+\frac{1}{2}\right)x}{2\sin\frac{x}{2}} = \frac{1}{2} + \sum_{k=1}^{n}\cos(kx)$$

the Dirichlet kernel.

(1) Please prove this identity. (5%)

(2) $D_n(x)$ is an even function of x. What is meant by an even function? (5%)

(3) For every n, we have

$$\frac{1}{\pi}\int_0^{\pi} D_n(x)dx = \frac{1}{2}$$

Please prove it. (5%)

(4) What is the period of $D_n(x)$? (5%)

4. A matrix A is called normal if

$$A\overline{A}^T = \overline{A}^T A$$

Suppose that A=B+iC, where B and C are real matrices. Show that A is normal if and only if BC = CB. (10%)

5. Solve the following initial value problems:

(a) $(D^2 + 4D + 5)y = 0 \qquad y(0) = 0 \qquad y'(0) = -3$ (10%)

(b) $(D^2 - 2D + \pi^2 + 1)y = 0 \qquad y(0) = 1 \qquad y'(0) = 1 - \pi$ (10%)

6. Find solution u(x,y) of the partial differential equation: (15%)

$u_x + u_y = 2(x+y)u$