國立交通大學 97 學年度碩士班考試入學試題

科目：工程數學(3031)　一般在職　考試日期：97年3月8日 第 1 節

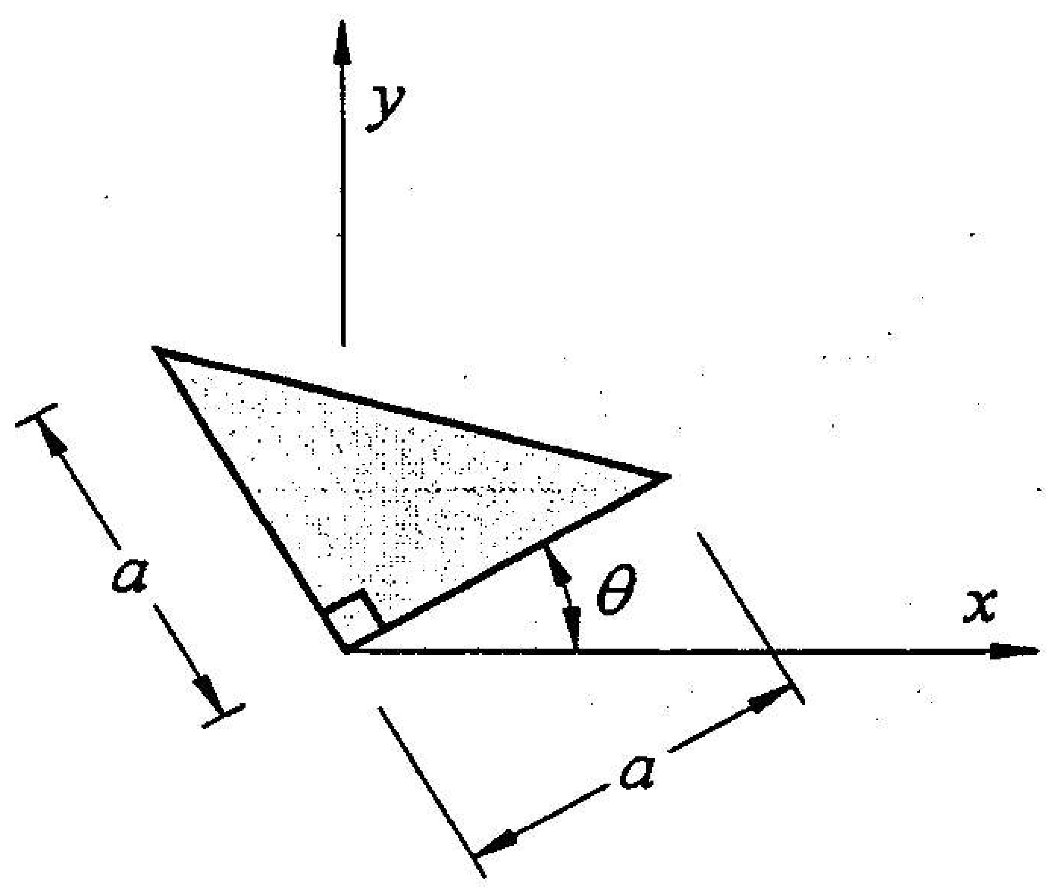
系所班別：機械工程學系　組別：機械系丙組　

【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. Evaluate the integral

$$\int_A x^2 dA, \ \int_A y^2 dA \ \text{and} \ \int_A xy dA$$

where A is the area of a right angle triangle as shown. (16%)



2. The system of differential equations is given by

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 3 \end{Bmatrix}$$

$$y_1(0) = 1, \ y_2(0) = -2, \ \dot{y}_1(0) = 2, \ \dot{y}_2(0) = 2$$

where $y_i = y_i(t)$, $\dot{y}_i = \dfrac{dy_i}{dt}$, $\ddot{y}_i = \dfrac{d^2 y_i}{dt^2}$, $i = 1, 2.$

Determine $y_i(t)$, $i = 1, 2.$ (17%)

3. (a). Consider the following one dimensional wave equation:

$$U_{tt} = c^2 U_{xx}$$

with boundary conditions: $U(0, t) = 0$, $U(L, t) = 0$ for all t and initial conditions: $U(x, 0) = f(x)$, $U_t(x, 0) = 0$.
The subscript $(\cdot)_t$ denotes partial derivative and c is wave speed.
Show that the solution of the above problem can be expressed as

$$U(x, t) = [f(x + ct) + f(x - ct)]/2$$ (10%)

(b). If $f(x) = \sin(x\pi/L)$ for $0 \leqq x \leqq L$, plot the diagrams of $f(x + ct)$, $f(x - ct)$, and $U(x, t)$ at $t = L/2c$ and L/c. (7%)

4. Please find the centroid of a hemispherical volume $x^2 + y^2 + z^2 \le 1,\ z > 0$ (17%)

5. Consider the differential equation

$$\frac{d^4 y}{dx^4} + \alpha^2 \frac{d^2 y}{dx^2} = 0, \qquad 0 < x < L,\ \alpha > 0$$

subject to boundary conditions

$$y = \frac{dy}{dx} = 0 \qquad \text{at } x = 0,$$

$$y = \frac{d^2 y}{dx^2} = 0 \qquad \text{at } x = L.$$

(a) Fine the general solution $y(x)$. (8%)

(b) Derive the characteristic equation in terms of α and L. Do not solve it. (9%)

6. The scalar function $\phi(x_1, x_2, x_3)$ is continuous, with continuous first partial derivatives in the interior V of smooth closed surface S. Let the unit vector $\mathbf{n} = n_1\mathbf{e}_1 + n_2\mathbf{e}_2 + n_3\mathbf{e}_3$ be outward normal to S, in which $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are the base vectors of a Cartesian coordinate system.

(a) Show that

$$\int_V \frac{\partial \phi}{\partial x_j} dV = \int_S \phi\, n_j\, dS, \qquad j = 1, 2, 3.$$

Note that it is not the Gauss theorem. (8%)

(b) Show that

$$\int_S x_i n_j\, dS = \begin{cases} \overline{V}, & i = j \\ 0, & i \neq j \end{cases}$$

where $\overline{V}$ is the volume enclosed by surface S. (8%)