資訊科學與工程，網路工程，多媒體工程，生醫影像與工程

國立交通大學 97 學年度碩士班考試入學試題

科目：線性代數與機率(1003)　　　　考試日期：97 年 3 月 9 日 第 2 節

系所班別：資訊系所跨組聯招　　組別：資訊聯招　　第 1 頁，共 3 頁

＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

線性代數

LINEAR ALGEBRA

There are six problems allotted a total of 50 points.

1. Let $A = \begin{pmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix}$ and S be the subspace of R^3 spanned by the column vectors of A.

(a) Find an orthonormal basis for $S^{\perp}$, the orthogonal complement of S (**3 points**).

(b) Let $P = A(A^TA)^{-1}A^T$, if $\mathbf{x} \in R^3$ and $\mathbf{x} \notin S \cup S^{\perp}$, show that $P\mathbf{x} \in S$ (**4 points**).

2. Consider the inner product space $C[0, 1]$ which is the set of all functions that have a continuous first order derivative on [0, 1]. The inner product of two functions $f(x)$ and $g(x)$ is defined by

$$\langle f, g\rangle = \int_0^1 f(x)g(x)dx.$$

(a) Use the Gram-Schmidt process to find an orthonormal basis E for the subspace S spanned by the vectors 1 and x^2 (**4 points**).

(b) Find the best least squares approximation to the function $\sqrt{x}$ on the interval [0, 1] by a function in S (**4 points**).

3. Define a linear transformation T from R^3 to R^3 as follows

$$T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\0\\1\end{bmatrix}; T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\\-1\end{bmatrix}; T\left(\begin{bmatrix}1\\0\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\-2\\1\end{bmatrix}.$$

(a) Find the matrix of the linear transformation T (**3 points**).

(b) Find a basis of the kernel of the transformation T (**3 points**).

(c) Find the coordinate vector of $T\left(\begin{bmatrix}2\\1\\2\end{bmatrix}\right)$ with respect to the basis = $\left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}\right)$ (**3 points**).

4.

(a) Consider a 3 x 2 matrix A and a 2 x 5 matrix B. How many possible dimensions of ker(AB) are there? What are they? You must justify your answers! (Note ker(AB) is the kernel of matrix AB) (**3 points**).

(b) Define a linear transformation T from R^{2x2} to R^{2x2} by $T(M) = M\begin{bmatrix}1&2\\0&1\end{bmatrix} - \begin{bmatrix}1&2\\0&1\end{bmatrix}M$. Find a basis of the image of T with respect to the standard basis

$$U = \left(\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\1&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}\right)$$ (3 points).

5. Students A and B were asked to solve the eigenvalues of the same matrix $M = \begin{bmatrix} a & b & c \\ 0 & d & 1 \\ 0 & 2 & e \end{bmatrix}$.

Unfortunately, Student A mistook the value of d and obtained the eigenvalues 0, 1, and 3. Student B mistook the value of e and obtained the eigenvalues 1, 1, and -2. Assume there were no other mistakes happened when they were solving the eigenvalues.

(a) Find the value of a **(3 points)**.

(b) Assume that the sum of correct eigenvalues of M is 1. Find the correct values of d and e **(6 points)**.

(c) Find the correct eigenvalues of M **(3 points)**.

6. Find a singular value decomposition for the matrix $M = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$ **(8 points)**.

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機率

7. (a) John throws 3 balls randomly into 5 boxes. What is the probability that a box contains at least 2 balls? (5 points)

(b) Let A and B be independent events with P(A)=2/3, P(B)=1/4. Find $P(A \cap B)$ and $P(A \cup B)$. (5 points)

8. Let X and Y be two independent random variables with E(X)=2, E(Y)=1 and Var(X)=3, Var(Y)=5.

(a) Find $E(3X - 2Y + 4)$. (3 points)

(b) Find $Var(3X - 2Y + 4)$. 3 points)

(c) Find Cov(X+Y, X-Y). (6 points)

9. Let X and Y be two independent random variables with the same density function given by

$$f(x) = \begin{cases} e^{-x} & 0 < x < \infty \\ 0 & otherwise. \end{cases}$$

(a) Find $P(X > 1)$. (3 points)

(b) Find $P(X > 2Y)$. (5 points)

(c) Find $P(\min(X, Y) < 1)$. (4 points)

10. The grades for a certain exam are normally distributed with mean 68 and variance 64. What percent of students get grades between 60 and 80. (Express your solution by using the standard normal distribution function Φ.) (6 points)

11. Let X be a nonnegative random variable. Prove that for any $t > 0$,

$$P(X \geq t) \leq \frac{E(X)}{t}.$$ (10 points)