

國立清華大學 命題紙

97 學年度 生命科學院 系(所) 丙 組碩士班入學考試

科目 微積分 科目代碼 0401 共 2 頁第 1 頁 *請在【答案卷】內作答

10%(1) (i) Show that $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$.

(ii) Evaluate $\int \tan^{-1} \sqrt{x} dx$.

10%(2) Evaluate $\int \frac{1}{x(x^2+2x+5)} dx$.

15%(3) The Gamma function is defined as follows

$$\Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} dx, \quad p > 0, \quad p \in \mathbb{R}.$$

Show that

(i) $\Gamma(p+1) = p\Gamma(p)$,

(ii) $\Gamma(n) = (n-1)!$, n is a positive integer,

(iii) $\Gamma(1/2) = \sqrt{\pi}$.

15%(4) Evaluate

$$\int_0^{1/2} \int_0^{\sqrt{1-x^2}} xy \sqrt{x^2+y^2} dy dx.$$

15%(5) Evaluate the following limits

(i) $\lim_{x \rightarrow +\infty} (\sin \sqrt{x+1} - \sin \sqrt{x})$,

(ii) $\lim_{n \rightarrow \infty} \left(\frac{n+x}{n-1} \right)^n$.

15%(6) (i) Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $r = \sqrt{x^2 + y^2 + z^2}$.

Show that $\nabla r^n = nr^{n-2}\vec{r}$ where $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$.

(ii) Let $f(x, y, z)$ be a differentiable function, $f: \mathbb{R}^3 \rightarrow \mathbb{R}$.

Define $\phi(t) = f(x+tx_0, y+ty_0, z+tz_0)$. Compute $\frac{d^2\phi}{dt^2}$

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10%(7) Assume $u(x, y)$ satisfies the following partial differential equation

$$(*) \quad a \frac{\partial u}{\partial x} + b \frac{\partial u}{\partial y} = 0, \quad a, b \in \mathbb{R}, \quad a^2 + b^2 \neq 0.$$

Let

$$\tilde{x} = ax + by,$$

$$\tilde{y} = bx - ay,$$

be a new coordinate. Write the equation (*) in terms of new coordinate and find the solution $u(x, y)$.

10%(8) Let $K = \{(x, y) : -1 \leq x \leq 1, -\pi \leq y \leq \pi\}$

$$f: K \rightarrow \mathbb{R}, \quad f(x, y) = xe^{-x} \cos y, \quad (x, y) \in K.$$

Find the largest and smallest values of $f(x, y)$, $(x, y) \in K$.