國　立　清　華　大　學　命　題　紙

97 學年度 生醫工程與環境科學系（所）丙(醫學物理與工程)組碩士班入學考試

科目 應用數學 科目代碼 2702 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. (10 %) Write the general solutions to

$$\mathbf{Ax}=\begin{bmatrix}1 & 2 & 2\\ 2 & 4 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}1\\ 4\end{bmatrix}$$

as the sum of a particular solution to **Ax=b** and the general solution to **Ax=0**.

2. (10%) Find the general solution of the following differential equation

$y''+xy'-y=e^{3x}$. Hint $e^{3x}=\sum_{k=0}^{\infty}\frac{3^k}{k!}x^k$.

3. (10 %) Given $P_n(x)=\sum_{m=0}^{M}(-1)^m\frac{(2n-2m)!}{2^n m!(n-m)!(n-2m)!}x^{n-2m}$, when n is even, $m=n/2$, else $m=(n-1)/2$ show that:

(a) $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]$

(b) $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$

4. (10%) (a) Compute the eigenvectors and eigenvalues of **A**.

(b) Is it possible to write **A** in the form $\mathbf{PDP}^{-1}$, where **D** is diagonal and **P** is invertible? If yes, what are **D** and **P**?

$$\mathbf{A}=\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 5\end{bmatrix}$$

5. (10%) Consider the Gamma function $\Gamma(\alpha)=\int_0^{\infty}e^{-\tau}\tau^{\alpha-1}d\tau,\quad \Gamma(\alpha+1)=\alpha\Gamma(\alpha)$ and the fact $\Gamma(1/2)=\sqrt{\pi}$. Find the Laplace transforms (a) $L\{t^{\gamma}\}$ and (b) $L\{t^{-1/2}\}$.

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6. (10 %) A mass attached to a spring is released from rest 1m below the equilibrium position ($y(0)=1,\ y'(0)=0$) for the mass-spring system and begins to vibrate. After $\pi/2$ seconds, the mass is struck by a hammer exerting an impulse on the mass. The system is governed by the initial value problem

$$y''+9y=-3\delta\left(t-\frac{\pi}{2}\right),\ y(0)=1,\ y'(0)=0$$

where $y(t)$ denotes the displacement from equilibrium at time t. Solve $y(t)$ and observe what happens to the mass after it is struck.

7. (10 %) Solve the initial value problem $\frac{dy}{dt}=1+y+t^2y+t^2,\quad y(0)=0$.

8. (10 %) Find the general solution to the following differential equation

$$x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y=x^{5/2},\quad x>0$$

9. (10 %) (a) Solve the initial value problem

$$y''+\omega^2y=\sin\gamma t,\quad y(0)=0, y'(0)=0, \omega\neq\gamma$$

(b) If $\omega\to\gamma$, what will be the solution?

10. (10 %) Given $J_{\nu-1}(x)+J_{\nu+1}(x)=\frac{2\nu}{x}J_\nu(x)$ and $J_{\nu-1}(x)-J_{\nu+1}(x)=2J'_\nu(x)$, show that

$$\frac{d}{dx}\left[xJ_\nu(x)J_{\nu+1}(x)\right]=x\left[J_\nu^2(x)-J_{\nu+1}^2(x)\right]$$