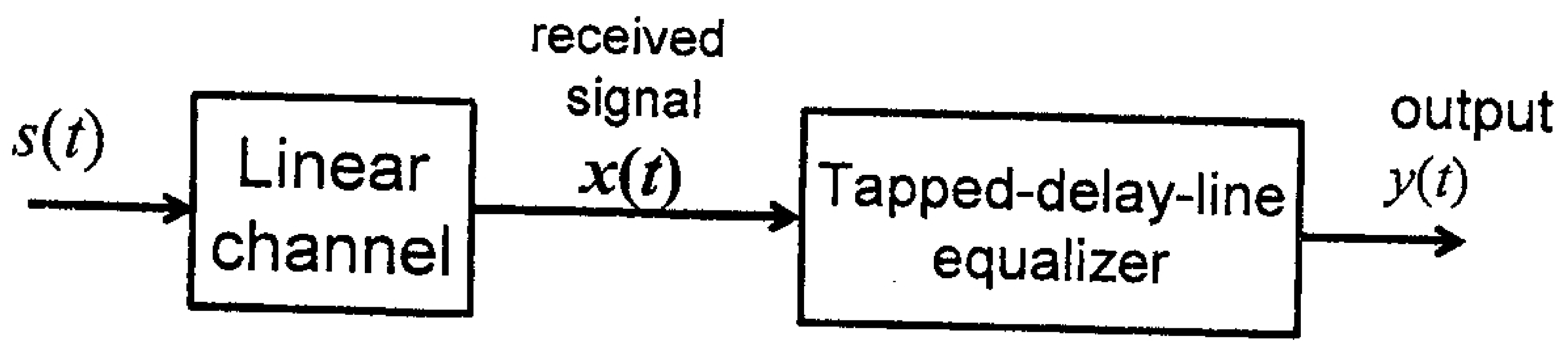
科目：通訊系統(5007)　　校系所組：中大通訊工程學系甲組

清大電機工程學系乙組、通訊工程研究所甲組

1. (20%) Consider the communication system in response to a signal $s(t)$ as in the following figure. The linear channel suffers from multipath distortion and the channel output is defined by

$$x(t) = a_1 s(t-\tau_1) + a_2 s(t-\tau_2)$$

where a_1 and a_2 are constant and τ_1 and τ_2 represent the corresponding delays of the propagation paths.



Now you are supposed to design the tapped-delay-line filter to equalize the multipath distortion produced by this channel. The time response of the tapped-delay-line filter is

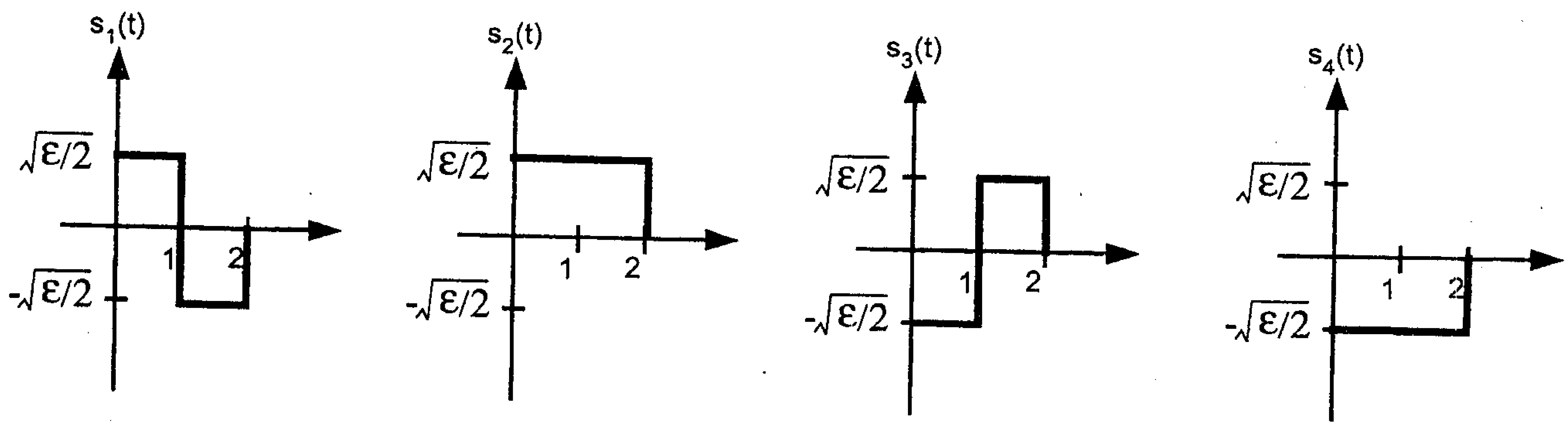
$$y(t) = w_0 x(t) + w_1 x(t-T) + w_2 x(t-2T).$$

(a) (4%) Find the frequency transfer function of the linear channel.

(b) (6%) Identify the desired frequency response at the equalizer output such that the channel multipath distortion is equalized.

(c) (10%) Assuming that $a_2 << a_1$ and $\tau_2 > \tau_1$, evaluate the parameters of the tapped-delay-line equalizer, i.e. w_0, w_1, w_2, and T, such that the channel multipath distortion is equalized.

2. (20%) Suppose we transmit a signal $s(t)$ through an additive white Gaussian noise channel. Assume that $s(t)$ is equal to $s_1(t)$, $s_2(t)$, $s_3(t)$, or $s_4(t)$ with equal probability, where the waveforms are given as follows:



(a) (6%) Use the Gram-Schmidt procedure in the order of $s_1(t)$, $s_2(t)$, $s_3(t)$, $s_4(t)$ to find the ordered set of orthonormal basis functions $\{f_1(t), f_2(t), \cdots, f_K(t)\}$, where K is the dimension of the signals. Find K and give the vector expression $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$, $\mathbf{s}_4$ for each of the waveforms, respectively, with the ordered set of basis functions derived above.

(b) (8%) Following from (a), suppose that the signal is passed through a vector channel so that the receiver observes

$$\mathbf{r} = \mathbf{C} \cdot \mathbf{s} + \mathbf{n},$$

where $\mathbf{n}$ is a Gaussian vector with zero mean and covariance matrix $\Sigma = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \cdots, \sigma_K^2)$. Derive the optimal detector that minimizes the error probability when $\mathbf{C} = \mathbf{1}$, where $\mathbf{1}$ is the K-by-K identity matrix, and compute the symbol error rate assuming that $\mathbf{C} = \mathbf{1}$ and $\sigma^2 \triangleq \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_K^2$.

(c) (6%) Following from (b), derive the symbol error probability for $\mathbf{C} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ and $\sigma^2 \triangleq \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_K^2$.

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3. (10%) Let $X(t)$ be a baseband transmitted signal of a symbol sequence A_n given by

$$X(t)=\sum_{n=-\infty}^{\infty} A_n g(t-T_d-nT)$$

where A_n is an independent identically distributed (iid) complex random sequence with zero mean and variance σ_A^2, T_d is a random variable uniformly distributed over [0, T], and $g(t)$ is a pulse shaping function. It is known that the power spectral density of $X(t)$ is given by

$$S_{XX}(f)=\frac{\sigma_A^2}{T}|G(f)|^2$$

where $G(f)$ is the Fourier transform of $g(t)$.

(a) (3%) Find $S_{XX}(f)$ if $g(t)=u(t)-u(t-T)$ where $u(t)$ is a unit-step function.

(b) (7%) Let $p(t)=u(t)-u(t-T_c)$ and

$$g(t)=\sum_{k=0}^{N-1} c_k p(t-kT_c)$$

where $T_c=T/N$ and c_k are iid binary random variables of $\{\pm 1\}$ with $\Pr(c_k=1)=\Pr(c_k=-1)=1/2$. Find $S_{XX}(f)$.

What are the distinctions between the results of part (a) and part (b)?

4. (10%) Consider a coherent binary frequency shift keying (BFSK) system where symbols '0' and '1' occur with equal probability. Let symbols '1' and '0' be encoded by signals $s_1(t)$ and $s_2(t)$, respectively, where

$$s_i(t)=\begin{cases}\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_i t), & 0\le t\le T_b\\ 0, & \text{otherwise}\end{cases}$$

in which E_b is the transmitted signal energy per bit, T_b is the symbol duration and $f_i=(n_c+i)/T_b$ for some fixed integer n_c. The received signal can be expressed as

$$x(t)=s_i(t)+w(t)$$

where $w(t)$ is a white Gaussian process with zero mean and power spectral density equal to $\mathcal{N}_0/2$.

(a) (5%) Determine the optimum receiver with minimum bit error rate (BER).

(b) (5%) Derive the BER of the optimum receiver in terms of the complementary error function or Q-function defined as follows:

$$\mathrm{Q}(u)=\int_u^{\infty}\frac{1}{\sqrt{2\pi}}e^{-z^2/2}\,dz=\frac{1}{2}\mathrm{erfc}\left(\frac{u}{\sqrt{2}}\right)$$

$$\mathrm{erfc}(u)=\frac{2}{\sqrt{\pi}}\int_u^{\infty}e^{-z^2}\,dz=2\mathrm{Q}(\sqrt{2}u)$$

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5. (20%) Consider the random variables X and Y with joint probability density function

$$f_{XY}(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{(x-A)^2+y^2}{2\sigma^2}}.$$

(a) (5%) Please find the marginal probability density function $f_Y(y)$.

(b) (15%) Please find the probability density function $f_R(r)$ of $R=\sqrt{X^2+Y^2}$ in terms of the modified Bessel function of the first kind of zero order $I_0(x)\equiv\frac{1}{2\pi}\int_0^{2\pi}e^{x\cos\theta}d\theta$.

6. (20%) We consider the maximum-ratio combining scheme. We have received a set of noisy signals $\{x_j(t)\}_{j=1}^{N}$, where $x_j(t)$ is defined by

$$x_j(t)=s_j(t)+n_j(t),\ j=1,2,\ldots,N.$$

The signal components $s_j(t)$ are locally coherent, that is, $s_j(t)=z_jm(t),\ j=1,2,\ldots,N$ where z_j are positive real numbers, and $m(t)$ denotes a message signal with unit power. The noise signals $n_j(t)$ have zero mean and variance σ_j^2, and they are statistically independent. The output of the linear combiner is defined by $x(t)=\sum_{j=1}^{N}\alpha_jx_j(t)$ where the parameters α_j are the combiner coefficients to be determined.

(a) (10%) Show that the output signal-to-noise ratio is $(SNR)_O=(\sum_{j=1}^{N}\alpha_jz_j)^2\Big/\sum_{j=1}^{N}\alpha_j^2\sigma_j^2$.

(b) (10%) Please show that the optimum values of the combiner's coefficients to maximize the output signal-to-noise ratio are $\alpha_j=z_j/\sigma_j^2$. (Hint: Schwarz inequality)